Multi-Stage Resource Allocation Under Asymmetric Information

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First Draft March 2010‡

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‡This represents very preliminary work. Please do not quote. Comments are welcome.

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I. Introduction

One of the most important determinants of a company's success is the effectiveness of its capital allocation decisions. For example, capital budgeting decisions are central to executing a firm's strategic plan. In addition, few decisions within the firms have as profound an impact on the promotion, compensation, and status of the managers vying for the allocation of that capital. This creates a conflict because the effectiveness of the firm's capital allocation decision depends critically on the information supplied by those self-interested managers. As a result, the analysis of the capital budgeting process was an early, and remains a continuing, subject of incentive contracting models (e.g., (Harris et al. 1982)).

This literature starts with the assumption that the individual managers vying for the capital allocations have better information about their ability to use the funds than does the person responsible for allocating the capital, and that the former will report their information in a self-interested manner. Beyond that, the literature makes many different assumptions as to the environment in which the capital allocation decision takes place. Much of the literature assumes that the capital budgeting process consists of a single stage in which:

- t = 0. the manager observes his private information
- t = 1. the firm offers the manager a contract which specifies how the resources will be allocated and how the manager will be compensated contingent on his report (see *t* = 2), the outcome of the use of the resources, and possibly other information
- t = 2. the manager reports his information and the capital is allocated as agreed upon. If not funded, the game ends with the manager being compensated as agreed upon
- t = 3. the manager invests some or all of the resources allocated to him
- t = 4. the outcome of the investment process and, possibly, other "general operating activities" in which the manager may have engaged in, are reported and the manager is compensated as agreed upon.

However, in many investment settings, the capital allocation decision is multi-stage. That is, based upon the manager's report about a project's potential, an initial investment is made, for example, an experiment is funded. Contingent on the outcome of this first-stage investment either an additional investment is made to fully implement the project or the project is abandoned. The capital allocation process may contain multiple stages for several reasons: additional information about the technology available (including its cost, reliability and scalability) or the market potential of the proposed new category of products can be acquired only after constructing a prototype of the plant or the product; or to satisfy regulatory obligations as with the multiple phases required for the introduction of new drugs. Thus, this multi-stage aspect of the investment process is more descriptive of investments in R&D or in new technology than of investments to expand existing capacity or to reduce costs when the technology involved is already known.

The purpose of this paper is to study the firm's capital allocation problem when the manager is privately informed and the capital allocation decision is made in two stages. The model under investigation consists of the following timeline:

Stage 1. The contracting and initial funding of the first (experiment) stage

- t = 0. the manager observes his private information
- t = 1. the firm offers the privately informed manager a contract which specifies the two-stage capital allocation scheme including: how the resources will be allocated and how the manager will be compensated contingent on the latter's report, the outcome of the use of the allocated resources and possibly other information
- t = 2. the manager reports his information and the first-stage experiment is funded or not as agreed upon. If first-stage experiment is funded the manager invests some or all of the resources, otherwise the game ends and the manager is compensated as agreed upon
- t = 3. the outcome of the first-stage experiment is publicly revealed

Stage 2. The full project implementation stage

- t = 4. based on the manager's report from t = 2 and the public information from t = 3, the full project is funded or not based on the agreed upon contract from t = 1. If the project is not implemented, the gamed ends and the manager is compensated as agreed upon
- t = 5. the results of the implemented project are publicly revealed and the manager is compensated as agreed upon.

As the above timeline highlights, we assume that the principal commits to a two-staged capital allocation scheme when designing the contract in t= 1. If, instead, the principal were to commit only to the first stage and choose a sequentially rational allocation scheme at stage 2, the game effectively reduces to a single-stage capital budgeting problem. We find that when the capital allocation decision is done in two-stages a number of the familiar results are overturned. For example, even though our model builds on (Antle and Eppen 1985) and the optimal investment rule in the first stage is a simple hurdle contract, the amount of capital that the principal allocates to fund the project in the first stage is not a constant but varies in the manager's report. Further, while the Antle-Eppen model predicts strict underinvestment, our model predicts both overinvestment or underinvestment in the second stage depending on the manager's report. Thus, capital allocation problems involving investments in R&D or new technology which require multi-stage investment decisions results in substantively different optimal processes than those involving non-R&D type investments which can be done in a single stage.

The paper is organized as follows. In the next section we review the relevant literature. In section III we present the model. We conclude and discuss extensions in section IV.

II. Literature Review

The present paper builds on the single-period "slack" model of (Antle and Eppen 1985) in which the revenue from the project is common knowledge, but the manager has private information about the minimum required investment for the first (i.e., experiment) stage of the investment project. Based on the manager's report about his private information, the principal decides whether to fund the first stage and, if so, the amount of the funding. The manager has an incentive to overstate the minimum required investment because he can consume any funding in excess of the amount needed to implement the project (i.e., the slack). A basic finding in this area is that the optimal capital budgeting process employs a hurdle where the principal funds any project whose reported minimum investment is less than some cut-off amount and the amount of funding is a constant independent of the report. Further, the hurdle is such that the firm turns down positive net present value projects in order to mitigate the manager's informational rents.[1,2]

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[1] (Bockem and Schiller 2009) is an exception in that it shows that if, in addition, the manager has to be motivated to acquire his private information, the range of funded projects will vary in his report. See also, (Kim 2006)

Subsequent work has proposed variations to the single-stage Antle-Eppen model which mitigate the residual inefficiency of the optimal budgeting process. For example, (Antle and Fellingham 1990), (Fellingham and Young 1990) and (Arya et al. 1994) extend the basic single-stage model to a repeated game in which each period different *independent* investment opportunities are discovered by the manager and reported to the principal who decides whether to fund each. They show that committing to a multi-period capital allocation process over a sequence of independent projects results in efficiency gains relative to treating each period separately. The sequencing of investment decisions helps to reduce the manager's information rent. In contrast, while the principal in our model also gains from commitment, we investigate a two-stage investment decision regarding a *single* project but in which an initial investment is an experiment whose outcome is informative about the return of the full investment opportunity in the second stage.

(Antle et al. 2006) and (Arya and Glover 2001) study the value of incorporating within the capital allocation process an option to delay the investment decision. The option gives the principal the ability to invest in a project discovered and reported on by the manager this period or to wait a period while the manager discovers and reports on an *entirely different and independent* project. In our model, rather than an option to delay the investment, the principal is endowed with an abandonment option in period 2 where at that point, based on the result of the stage 1 investment, the principal can choose to either fully fund the project or abandon it. The value of an abandonment option is also studied by (Arya and Glover 2003). However, they pose the problem within a pure moral hazard problem that does not involve any investment by the principal but does have the principal privately observing the intermediate information on which his abandonment decision will be based.

As already noted, in our model the manager's incentive to misreport his cost observation is that he can consume any resources which he receives from the principal in excess of the amount necessary to fund the investment project. Another branch of the capital allocation literature motivates the incentive problem by assuming the manager has two responsibilities.[3] As in the models above, the manager is responsible for discovering and reporting on investment

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[2] The optimality of hurdle rates has also been shown in capital budgeting problems without incentive conflicts, e.g., (Baldwin 1982) and (Prastacos 1983)

[3] A third branch is one that assumes that the manager has private benefits of control; for example, the manager receives utility from managing larger rather than smaller investment projects. See, for example, (Baldenius 2003). A fourth branch assumes that, in addition to the manager's incentive problem arising from his private information, the principal has an incentive problem arising from the incompleteness of contracting. See, (Baiman and Rajan 1995).

opportunities. In addition, the manager exerts a “general purpose” effort, for which he incurs disutility and which results in an additional cash flow to the firm which is independently distributed of the cash flow generated by the investment. Only total cash flow from the implemented project and the general purpose effort is reported. As a result, the principal cannot tell how much of the total was produced by the investment versus the manager’s effort. Thus, the manager has an incentive to understate the quality of the investment in order to hide the effect of shirking in his general purpose effort (the “shirking” model). The principal controls the resulting inefficiency by choosing how to compensate the manager and by distorting the capital allocation rule (see, for example (Dutta and Reichelstein 2002) and (Dutta 2003)) . As in the slack models, a consistent feature of these shirking models is that the principal finds it optimal to ration capital. However, as with our analysis (Dutta and Fan 2009) find situations under which it is optimal for a firm to set the investment hurdle rate at less than its cost of capital. In their model the manager’s productive effort improves the quality of any investment project. In order to encourage this effort and truthful reporting, they show that there exist problem *parameters* such that the firm’s chosen hurdle rate is less than the firm’s cost of capital and hence the firm overinvests. Although our model is considerably different, we find a somewhat similar result in that *regardless of the problem parameters* the optimal capital allocation process will involve overinvestment in the second stage for certain of the manager’s first stage reports.

Most closely related to our work is (Pfeiffer and Schneider 2007) who also model a multi-stage capital allocation problem in which the principal has an abandonment option. Our models are similar in that the manager enjoys his informational rent only if the project is completed. However, our models differ in a number of ways. One difference is that in Pfeiffer and Schneider the manager privately observes the results of the first-stage investment and must be motivated to report the information to the principal. In our model, the outcome of the first-stage investment is public and contractible information. A second major difference between the two models is the source of the manager’s information rent and the principal’s benefit from abandoning the project at stage 2. Pfeiffer and Schneider assume a “shirking” model in which the manager’s general purpose effort produces a deterministic cash flow. As a result, the principal has two benefits from abandoning the project at stage 2. First as is normal, the principal avoids investing in bad projects. Second and most important for their results, by abandoning the project the principal perfectly learns the manager’s general purpose effort and therefore both avoids paying the information rent and achieves first-best with respect to the manager’s effort. Thus, there is no deadweight loss incurred by abandoning the project. In essence, abandoning the project is the cost of perfectly monitoring the manager’s effort. In contrast, in our model if the project is abandoned at stage 2, the stage 1 investment has no value and hence the principal cannot recoup the information rent. Further, the manager can only consume the information rent (i.e., perquisites) if the project is funded at stage 2. Thus, the abandonment decision entails a deadweight loss in that the information rent incurred by the principal may not equal the information rent consumed by the manager. As a result of these differences, our major results are quite difference from those of (Pfeiffer and Schneider 2007).[4]

III. Model

We extend the (Antle and Eppen 1985) capital budgeting model to study the capital allocation decision for a two-stage investment project. The two stages can be thought of as a research or experiment stage and a project implementation stage where the research stage provides information about the feasibility and profitability of implementing the project. A risk-neutral firm (the principal) hires a risk-neutral manager to manage both stages of the project. At *t* = 1 it is common knowledge that if the research stage is successful and the project is implemented it will generate gross cash flow of R at *t* = 4. However, at the time that the manager is hired, he privately knows the minimum amount of funding required to carry out the research stage. What is uncertain to both the manager and the principal at *t* = 1 is the additional amount of funding required to implement the project if the experiment does succeed. We assume that the research stage requires a minimum investment of $c_i = (c_1,...,c_N)$ to be carried out. Any investment less than c_i will cause the research stage to fail with certainty while any investment of c_i or more will assure that the research stage succeeds with probability π. This minimum investment is privately observed by the manager at *t*=0. The information produced by the research stage is then two-fold: whether the research has been successful and hence the project will succeed if implemented and, if the first result is positive, the additional investment required to implement it. The latter is represented by the continuous variable $m \in [0, M]$. The results of the research stage are publicly observed at *t* = 3. The probability densities of c and m are common knowledge and represented by $p(c_i)$ and $p(m)$ respectively.

At *t* = 2, based on his private information, the manager submits a report $\hat{c}_i$ to the principal about the required minimum research investment. The latter then decides whether to fund the research stage and the amount of the research budget $b(\hat{c}_i)$ to provide to the manager. After receiving the budget, the manager invests the necessary capital to fund the research stage. The

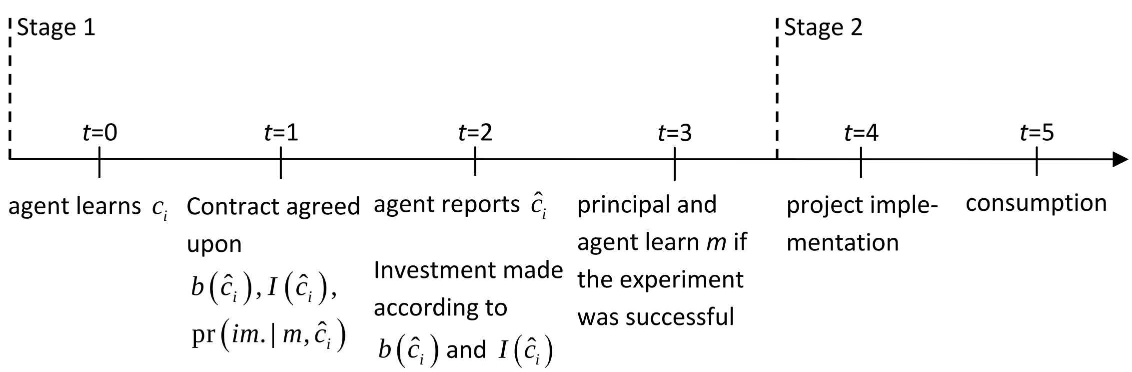
[4] Using a somewhat similar model to (Pfeiffer and Schneider 2007), (Johnson et al. 2010) extend their analysis to the investment decision by a firm in assets which may be used by multiple divisions. For a related paper, see also, (Vaysman 2006).

manager's investment is not observable to the principal. Since the probability that the research stage will succeed is π for all investments by the manager of c_i or greater, the manager will invest exactly c_i (if $b(\hat{c}_i) > c_i$) in the research and invest the remaining funds $b(\hat{c}_i) - c_i$ in potential slack or perquisites.

If the research stage fails, the game ends and all invested funds are lost, including any potential slack in which the manager has invested. If the research stage succeeds but the principal decides not to implement the project, the game ends and all invested funds are lost, including any potential slack in which the manager has invested. However, if the research stage succeeds and the principal invests the necessary additional funds to implement the project, the principal consumes the returns of the project net of his payments, $R - b(\hat{c}_i) - m$, and the manager consumes the potential slack $b(\hat{c}_i) - c_i$.[5]

We assume that at *t* = 1 the principal can commit to a funding rule for both stages of the project. Specifically, at *t* = 1 the principal offers the manager a contract which specifies: the research approval decision rule $I(\hat{c}_i) \in \{0,1\}$, the research budgeting rule $b(\hat{c}_i)$, and the project implementation rule given that the research stage succeeds $\mathrm{pr}(im. \mid m, \hat{c}_i) \in [0,1]$. The timing of the model is depicted in Figure 1.

Figure 1: Timeline



The manager's expected utility after observing the required investment in the research stage c_i and reporting $\hat{c}_i$, is given by

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[5] (Bolton and Dewatripont 1994) make a similar assumption that the agent receives private benefits from a project only if it is not canceled by the debt holders.

$$U_A = \pi \int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, \hat{c}_i\right) p\left(m\right) I\left(\hat{c}_i\right)\left(b\left(\hat{c}_i\right) - c_i\right).$$

Note that the manager's expected utility depends on the principal's choice of probabilities that the project is implemented. This arises from the assumption that the manager can only consume a project's slack if it is actually implemented.

We use the Revelation Principle to restrict our analysis to contracts which induce the manager to honestly report his private observation of the required minimum investment. In order to simplify notation we will henceforth write $b_i \equiv b\left(c_i\right)$. At t = 1 the principal's problem is to design a contract which maximizes her expected utility

$$U_P = \sum_{i=1}^{N}\left[-b_i + \pi \int_{m=0}^{M} \left(R - m\right) \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right)\right] I\left(\hat{c}_i\right) p\left(c_i\right)$$

subject to following constraints

(1) $$\pi \int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right)\left(b_i - c_i\right) \geq \pi \int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_j\right) p\left(m\right)\left(b_j - c_i\right)$$

$\forall c_i, c_j$ such that $I\left(c_i\right) = I\left(c_j\right) = 1$,

(2) $$\pi \int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right)\left(b_i - c_i\right) \geq 0$$

$\forall c_i$ such that $I\left(c_i\right) = 1$,

(3) $$0 \geq \pi \int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_j\right) p\left(m\right)\left(b_j - c_i\right)$$

$\forall c_i, c_j$ such that $I\left(c_i\right) = 0$ and $I\left(c_j\right) = 1$.

Constraint (1) ensures that the manager who observes signal c_i, for which the research stage would be funded, truthfully reports his observation rather than claim a different signal $c_j \neq c_i$, which would also lead to the research stage being funded. Constraint (2) plays two roles. It ensures that the manager who observes signal c_i, for which the research stage would be funded, truthfully reports his observation rather than claim a different signal $c_j \neq c_i$ which would lead to the research stage not being funded and the game ending. We assume that the manager's outside opportunity wage is zero, and therefore Constraint (2) also represents the manager's ex ante Individual Rationality constraint. Constraint (3) ensures that a manager who observes c_i, for which the research stage would not be funded doesn't report an observation $c_j \neq c_i$ for which the principal would provide research funding.[6]

We begin the analysis of the principal's problem by characterizing the optimal stage-1 (Proposition 1) and stage-2 (Proposition 2) funding rules.

Proposition 1:[7]

(i) The optimal funding rule for the stage-1 research activity specifies a cut-off value c_h such that $I(c_i)=1$ for $c_i \leq c_h$ and $I(c_i)=0$ for $c_i > c_h$.

(ii) The research budget is weakly increasing in the manager's private information c_i.

Proposition 1 (i) establishes that, as in the one-stage Antle-Eppen model, the principal commits to develop all projects which require a minimum investment that falls below a threshold. In the one-stage problem, this threshold is chosen such that not all profitable projects are funded in order to limit the manager's slack – there is capital rationing. Part (ii) establishes a result which is different from the one-stage problem: the budget granted to the manager may not be a constant. This difference arises because in our two-stage the principal has an additional lever of control, the second-stage implementation rule.[8] When submitting the report, the manager is interested in maximizing the expected amount of budgetary slack *which he consumes*. In our model, this depends on the *second-stage* funding rule. By basing the second-stage funding rule on the manager's report c_i, the principal may be able to induce truth-telling in the first stage more efficiently.[9] Specifically, decreasing the probability that a high stage-1 required investment project is implemented, even if it succeeded at the research stage, will make it less

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[6] Without loss of generality, we ignore the possibility that the principal also compensates the manager when: (a) the principal does not invest in the research stage, (b) the research stage was not successful, and (c) the research stage was successful but the principal chooses not implement the project. The proof of this assertion is relatively long but straight-forward and is available upon request.

[7] All proofs are in the Appendix.

[8] We later show that, in fact, the budget is strictly increasing in the reported investment requirement.

[9] The principal can always choose funding rules which mimic a one-stage investment by defining a second-stage rule that is independent of the agent's report.

desirable for the manage to report this high investment requirement. This would allow the principal to reduce the budget for low reports.

While Proposition 1 examined the principal's optimal choice in the first stage, Proposition 2 investigates the optimal funding rule for the second stage.

Proposition 2:

(i) The optimal probability of funding at the second stage is weakly decreasing in the manager's private information c_i.

(ii) The optimal funding rule for the second stage is a cut-off value $\overline{m}_i$ such that $\mathrm{pr}\left(im. \mid m, c_i\right) = 1$ for $m \leq \overline{m}_i$ and $\mathrm{pr}\left(im. \mid m, c_i\right) = 0$ for $m > \overline{m}_i$.

Part (i) of Proposition 2 supports the previously discussed idea that the principal may find it optimal to base the probability of funding in the second stage on the manager's first-stage report in order to mitigate the manager's incentives for dishonest reporting. This true even though c_i and m are independently distributed. For example, the principal could decrease the probability of funding at the second stage for higher stage-1 reports, making it less attractive to the manager to overstate his observed signal. That is, by overstating his observation the manager may increase the stage 1 funding and thereby his *potential* slack, $b_i - c_i$, but decrease the probability of the project being implemented and therefore decrease the probability of actually consuming that slack. This is very similar to the trade-off faced by a bidder in a first-price auction. Of course the cost to the principal of doing so is an inefficient second-stage funding decision, both not funding projects that would be ex post efficient to fund and/or funding projects that would be ex post inefficient to fund. Part (ii) shows that the principal's funding decision rule at the second stage is a hurdle contract as at the first stage, but which depends on both the manager's stage-1 report and the additional investment report produced by the stage-1 experiment.

Corollary 1 establishes additional characteristics of the optimal contract.

Corollary 1:

(i) The manager's expected rent is strictly decreasing in observations for which the research stage will be funded.

(ii) The Individual Rationality constraint for the manager with the highest minimum required observation for which the research stage will be funded is binding. All other Individual Rationality constraints are not binding.

(iii) Constraint (3) is redundant.

(iv) The global truth-telling constraints (1) for can be reduced to only those that involve reporting the adjacent higher observation.

Corollary 1 are standard results in the literature on capital allocation, and more generally, adverse selection. The manager's expected rent is a function of both his potential rent, $b_i - c_i$, and the likelihood of the project being implemented, $\int_{m=0}^{\bar{m}_i} p(m)$. From (1) it is easy to see that $\int_{m=0}^{\bar{m}_i} p(m) b_i \geq \int_{m=0}^{\bar{m}_j} p(m) b_j$ for $c_i < c_j$. Therefore, consistent with our earlier discussion, while the principal grants a larger budget for a larger reported minimum investment observation, thereby encouraging the manager to over report, the decrease in the probability of implementing the project at the second stage mitigates this incentive to over-report.

Given the previous results, we can restate the principal's maximization problem as follows

$$U_P = \sum_{i=1}^{N} \left[-b_i + \pi \int_{m=0}^{\bar{m}_h} (R-m) p(m) \right] p(c_i)$$

s.t.

(1.2) $\pi(b_i - c_i) \int_{m=0}^{\bar{m}_i} \text{prob}(m) = \pi(b_{i+1} - c_i) \int_{m=0}^{\bar{m}_{i+1}} \text{prob}(m) \qquad \forall i+1 \leq h$

(2.2) $b_h = c_h$

The principal now has to choose cutoff values for the research and implementation stages as well as budget rule for the initial research funding. While the first is a fixed value, the latter two can be conditioned on both the manager's report (as indicated by the indices) as well as on the result of the research stage.

Corollary 2: The optimal research budget rule is:

$$b_i = c_i + \frac{\sum_{k=i+1}^{h} \left(c_k - c_{k-1}\right) \int_0^{\overline{m}_k} \mathrm{p}(m)}{\int_0^{\overline{m}_i} \mathrm{p}(m)}.$$

Corollary 2 shows that the optimal budget b_i represents the manager's virtual minimum required investment, which is determined by his true minimum required funding observation (c_i) plus the manager's rent $\left(\frac{\sum_{k=i+1}^{h} \left(c_k - c_{k-1}\right) \int_0^{\overline{m}_k} \mathrm{p}(m)}{\int_0^{\overline{m}_i} \mathrm{p}(m)} \right)$. If the principal were not able to commit to a second stage funding rule and, hence, acted sequentially rationally, she would choose a constant second stage cutoff value, $\overline{m}_i = R \; \forall i$, in which case $\int_0^{\overline{m}_k} \mathrm{p}(m) = \int_0^{R} \mathrm{p}(m) \; \forall k$. Then the optimal research budget becomes $b_i = c_i + \sum_{k=i+1}^{h} \left(c_k - c_{k-1}\right) = c_h$, which is the solution to the Antle-Eppen model. This highlights the fact that if we find that the optimal budgets vary in the manager's report it is because of the principal's ability to commit to the second stage investment decision.

Before discussing the reason for this we first establish that the stage-1 budget is *strictly* increasing in the manager's report while the stage-2 funding rule is *strictly* decreasing in the manager's report. To do so, we assume that $\Delta = c_i - c_{i-1} \quad \forall c_i, c_{i-1} \in \left(c_1, \ldots, c_N\right)$, and that $\mathrm{p}(c_i) = \frac{1}{N}$ and $\mathrm{p}(m) = \frac{1}{M}$. In this case, the optimal budget simplifies to $b_i = c_i + \Delta \sum_{k=i+1}^{h} \overline{m}_k \Big/ \overline{m}_i$. Proposition 3 then further characterizes the optimal stage 1 and stage 2 funding rules.

Proposition 3:

(i) the second stage cutoff value $\overline{m}_i$ is *strictly* decreasing in c_i, and

(ii) the optimal research budget is *strictly* increasing in c_i.

Proposition 3 (ii) shows more clearly that our results deviate from those of the one-stage Antle-Eppen model. By allowing the principal to include a second instrument in the contract, the optimal budget is not a constant but is strictly increasing in the reported minimal required investment. In the Antle-Eppen model, the principal's optimal reaction to the manager's report is either to not fund the project or to provide a budget which covers the cost of the hurdle project (c_h in our case). Instead in our model, the principal fine tunes both the stage-1 and stage-2 funding rules. The reason why the principal is better off doing this in our model is because the amount used to fund the research stage is a deadweight loss if the project is not implemented. It benefits neither the principal nor the manager because the principal cannot recoup the stage-1 investment if the project is not implemented and the manager only consumes the slack for implemented projects. The principal can reduce this deadweight loss but still induce truth-telling by reducing b_i from b_i^{AE} but distorting the stage-2 investment decision.[10]

In addition to reducing the deadweight loss, the above results show that the principal can use the presence of the second stage to reduce the manager's expected slack. In (Antle and Eppen 1985) a manager with cost c_i receives expected slack $s_i^{AE} = c_h - c_i = \sum_{k=i+1}^{h} \Delta$. In our model the manager's expected slack is $s_i = \frac{\overline{m}_i}{M}(b_i - c_i) = \sum_{k=i+1}^{h}\left(\Delta \frac{\overline{m}_k}{M}\right) < s_i^{AE}$. So while the manager's slack in (Antle and Eppen 1985) increases by Δ for the next lower cost observation (i.e. i instead of $i+1$), it only increases by $\Delta \frac{\overline{m}_{i+1}}{M}$ in our model.

Antle and Fellingham (1990) derive a somewhat similar stage-1 funding result, but only for specific parameters. As described in the Literature Review, in their model a manager is hired to sequentially oversee two independent projects. This allows the principal to provide the manager's total slack in the second period. If this instrument relaxes the underinvestment problem, the principal could continue to fund all projects above the one-period hurdle with the same constant budget and fund the project immediately below the hurdle rate with a smaller budget. Adding the second round of investing in Antle-Fellingham produces an optimal stage-1 funding rule which divides the set of projects into three categories: those that aren't funded, those that are and receive the same budget, and that one which is funded and receives a

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[10] We provide more discussion of the induced distortion below.

smaller budget than the others. Our two-stage investment process allows us to more finely tune the stage-1 funding rule.

Proposition 4 compares our results for $\overline{m}_i$ with the sequentially rational behavior of the principal.

Proposition 4: The optimal second stage cutoff value results in

(i) overinvestment for the lowest minimum required research investment, and

(ii) underinvestment for the highest minimum required research investment.

Ex post efficiency requires that $\overline{m}_i = R \ \forall i \le h$. However, Proposition 3 shows that $\overline{m}_1 > R > \overline{m}_h$, independent of the chosen stage-1 research budgets. This means that the principal induces distortion both for the most efficient (c_1) and the least efficient managers (c_h). It also implies that if there is a required investment level c_i for which the principal chooses not to induce a stage-2 distortion, it will be at an intermediate level. Proposition 4 stands in contrast to the standard results in the literature on adverse selection where the principal usually finds it optimal to not distort production for the most efficient type (c_1 in our case).[11] In order to understand the reason for this difference, recall that in the usual adverse selection model, the rent extracted from the principal is the rent consumed by the manager (as in a pure exchange model). In our model however, there is the possibility of a deadweight loss. This occurs because while the principal incurs the cost b_i whenever the research stage is funded, the resulting slack of $b_i - c_i$ is consumed by the manager only if the project is implemented. The b_i is a deadweight loss if the principal decides not to implement the project at the second stage (which happens with probability $1 - \overline{m}_i / M$). Thus in our model the principal is trading off not just production efficiency and informational rents, *but also* deadweight loss.[12]

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[11] This is also the standard result in the capital budgeting literature. For example Dutta and Reichelstein (2002) derive the result that only a manager with the best possible investment project receives the same incentives as in the case without information asymmetry about the project's quality.

[12] Trading off productive efficiency and deadweight loss, although not informational rent arises in the implicit contracting literature (e.g., MacLeod 2003 and Rajan Reichelstein 2009).

Note that for the more efficient managers (lower c_i's) the principal can increase $\overline{m}_i$ and decrease b_i, keeping the manager's incentives and expected rent fixed while decreasing the deadweight loss. Of course, that decrease in deadweight loss has to be traded off against any inefficient stage-2 investment. Note that to decrease the total amount of rents she has to pay, the principal commits to forego implementation of profitable projects for high development cost observations. To see this, recall the manager's Truth-telling Constraint, $\pi(b_i - c_i)\overline{m}_i / M = \pi(b_{i+1} - c_i)\overline{m}_{i+1} / M$. By decreasing $\overline{m}_h$ the principal not only decreases the desirability of reporting c_h, but also reduces the manager's budget for all lower observations. This is essentially the same result as in the Antle-Eppen model. One can also see this by examining the optimal budget $b_i = c_i + \Delta \sum_{k=i+1}^{h} \overline{m}_k \Big/ \overline{m}_i$. Note that $\overline{m}_h$ appears in the numerator of every funded research budget. Therefore, reducing $\overline{m}_h$ reduces every funded research budget.

IV. Conclusion

Many capital investment decisions, especially those involving investing in R&D or investments involving new technology, are done in multiple stages. The reason is that additional information about the technology available (including its cost, reliability and scalability) or the market potential of the proposed new category of products can often be acquired only after producing a prototype of the plant or the product; or to satisfy regulatory obligations as with the multiple phases required for the introduction of new drugs. Such investments are particularly prone to asymmetric information. In this paper we examine how the optimal capital budgeting process changes as a result of the multiple stages.

We base our analysis on a model in which the incentive problem arises because the manager privately observes the minimum required investment for the first (experiment) stage and he can consume any funding in excess of the minimum required. If the manager cannot consume this slack unless the project is ultimately funded, we find that the optimal capital budgeting solution for the two-stage process is quite different from that for the one-stage process. The presence of a second-stage funding decision provides the principal an additional lever of control with which to influence the behavior of the manager. In a one-stage game, the principal finds it optimal to either fund a project at a fixed level or not fund it at all. With the second-stage funding decision, the principal finds it optimal to vary the amount of the stage-1 funding in the manager's report. Further, the principal also finds it optimal to distort his second-stage funding decision in order to control the manager's stage-1 incentives. In particular, the principal finds it optimal to both overinvest and underinvest, relative to the sequentially rational investment level, depending on the manager's stage-1 observation. The present paper represents preliminary work in this area. We plan on pursuing a number of extensions. First, we need to further characterize the optimal solution. For example, we need to characterize the optimal stage-1 funding to establish whether the amount of capital rationing in the first stage is affected by introducing an abandonment option. Second, we would like to better understand how the optimal capital allocation process varies in the uncertainty of the environment, and therefore in the severity of the agency problem. Finally, we have assumed that the manager is exogenously endowed with the minimum required investment realization. An interesting extension would be to examine how the solution changes if the manager has to be motivated to acquire that information.

Appendix

Proof of Proposition 1:

(i) Constraints (2) and (3) imply

$$\int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right)\left(b_i - c_i\right) \geq \int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right)\left(b_i - c_j\right) \; \forall \; I\left(c_i\right) = 1, I\left(c_j\right) = 0$$

which only holds if $c_i \leq c_j \; \forall \; I\left(c_i\right) = 1, I\left(c_j\right) = 0$.

(ii) Constraint (1) implies that: $\dfrac{b_i - c_i}{b_j - c_i} \geq \dfrac{\int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_j\right) p\left(m\right)}{\int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right)}$.

Constraint (1) also prevents the manager from reporting c_i if he observes c_j, which implies: $\dfrac{\int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_j\right) p\left(m\right)}{\int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right)} \geq \dfrac{b_i - c_j}{b_j - c_j}$.

Combining the two yields $\dfrac{b_i - c_i}{b_j - c_i} \geq \dfrac{b_i - c_j}{b_j - c_j}$, which, establishes that $c_j \geq c_i \Rightarrow b_j \geq b_i$. ■

Proof of Proposition 2:

(i) From (1) we know $\dfrac{\int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right)}{\int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_j\right) p\left(m\right)} \geq \dfrac{b_j - c_i}{b_i - c_i}$. Proposition 1 (ii) established that, $\forall c_j \geq c_i$ such that $I\left(c_i\right) = I\left(c_j\right) = 1$, the right hand side of the inequality is greater than or equal to one, which implies that $\int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_i\right) p\left(m\right) \geq \int_{m=0}^{M} \mathrm{pr}\left(im. \mid m, c_j\right) p\left(m\right)$. This establishes that the probability of stage-2 funding is decreasing in the manager's stage-1 report.

(ii) We first show, that the Lagrangian of the principal's decision problem is linear in the probability of funding in the second stage which proves that the principal will choose $\mathrm{pr}\left(im.|\,m,c_j\right)\in\{0,1\}$. In the second step, we show that the term which determines whether the probability is one or zero is decreasing in m .

The Lagrangian for the principal's maximization problem is given by

$$L=\sum_{i=1}^{h}\left[-b_i+\pi\int_{m=0}^{M}\left(R-m\right)\mathrm{pr}\left(im.|\,m,c_i\right)p\left(m\right)\right]p\left(c_i\right)$$
$$+\sum_{i=1}^{h}\sum_{j=1}^{h}\lambda^{ij}\left\{\pi\int_{m=0}^{M}\mathrm{pr}\left(im.|\,m,c_i\right)p\left(m\right)\left(b_i-c_i\right)-\pi\int_{m=0}^{M}\mathrm{pr}\left(im.|\,m,c_j\right)p\left(m\right)\left(b_j-c_i\right)\right\}$$
$$+\sum_{i=1}^{h}\gamma^{i}\pi\int_{m=0}^{M}\mathrm{pr}\left(im.|\,m,c_i\right)p\left(m\right)\left(b_i-c_i\right)-\sum_{i=1}^{h}\sum_{j=h+1}^{N}\mu^{ji}\pi\int_{m=0}^{M}\mathrm{pr}\left(im.|\,m,c_i\right)p\left(m\right)\left(b_i-c_j\right),$$

where λ^{ij} , γ^{i} , and μ^{ji} are the respective Lagrange multipliers for constraints (1), (2), and (3), respectively. The principal will commit to a funding rule $\mathrm{pr}\left(im.|\,m,c_i\right)$ for each possible realizations of m and c_i . Hence, for each $\left(m,c_i\right)$ realization she will maximize the following:

$$L_{m,i}=\mathrm{pr}\left(im.|\,m,c_i\right)\pi\, p\left(m\right)A+B$$

where *B* are terms not involving $\mathrm{pr}\left(im.|\,m,c_i\right)$ and $A=\left(R-m\right)p\left(c_i\right)+\sum_{j=1}^{h}\lambda^{ij}\left(b_i-c_i\right)-\sum_{j=1}^{h}\lambda^{ji}\left(b_i-c_j\right)+\gamma^{i}\left(b_i-c_i\right)-\sum_{j=h+1}^{N}\mu^{ji}\left(b_i-c_j\right)$. Note that $L_{m,i}$ is linear in $\mathrm{pr}\left(im.|\,m,c_i\right)$. Hence, $\mathrm{pr}\left(im.|\,m,c_i\right)=1$ for $A\geq 0$ and $\mathrm{pr}\left(im.|\,m,c_i\right)=0$, otherwise. Further, note that A is decreasing in m . ■

Proof of Corollary 1:

(i) Using the result from Proposition 2 (i) we get $\int_{m=0}^{\bar{m}_i}p\left(m\right)\left(b_i-c_i\right)\geq\int_{m=0}^{\bar{m}_j}p\left(m\right)\left(b_j-c_i\right)>\int_{m=0}^{\bar{m}_j}p\left(m\right)\left(b_j-c_j\right)$ for $c_i<c_j\leq c_h$, where the first inequality comes from (1) and the second inequality from $c_i<c_j$.

(ii) If (ii) were not true, the principal could reduce b_h thereby increasing her objective function without violating any constraints.

(iii) A sufficient condition for Constraint (3) to hold is that $0 \geq \pi \int_{m=0}^{\bar{m}_h} p(m)(b_h - c_{h+1})$, which is clearly true given (ii).

(iv) Define $T_i = b_i \int_{m=0}^{\bar{m}_i} p(m)$ and $v_i = \int_{m=0}^{\bar{m}_i} p(m)$. For the remaining analysis assume $c_i < c_j < c_k \leq c_h$. To prevent the manager from reporting a lower type $T_j - v_j c_j \geq T_i - c_j v_i$ has to hold, which implies

(A) $c_k(v_i - v_j) \geq c_j(v_i - v_j) \geq T_i - T_j$.

This implies

(B) $T_k - c_k v_k \geq T_j - c_k v_j \geq T_i - c_k v_i$

where the first inequality follows from (1) and the second follows from (A). (B) in turn implies that if the manager doesn't have incentives to report a lower observation, he also will not report any even lower. The same analysis holds for reporting higher observations and, hence, only the adjacent truth-telling constraints need to be considered.

Next, we show by contradiction that the adjacent upwards truth-telling conditions are binding. Assume the opposite: $T_j - v_j c_j > T_{j+1} - v_{j+1} c_j$. In this case, the principal could lower all T_i for $i \leq j$ by a constant so as to make the stated constraint binding (which does not affect (2) as this only binds for the highest observation).

Next we establish that the binding upward adjacent constraints make the downwards constraints redundant. If $T_i - v_i c_i = T_{i+1} - v_{i+1} c_i$, then $T_{i+1} - T_i = c_i(v_{i+1} - v_i) \geq c_{i+1}(v_{i+1} - v_i)$ or $T_{i+1} - v_{i+1} c_{i+1} \geq T_i - v_i c_{i+1}$ where, again, the equality follows from the adjacent upward constraint being binding and the inequality from $c_{i+1} > c_i$ and therefore $v_i \geq v_{i+1}$. ■

<u>Proof of Corollary 2:</u>

Solving (1.2) for b_{h-1} and recursively for the lower budgets yields the result. ■

Proof of Proposition 3:

(i) the second stage cutoff value $\bar{m}_i$ is *strictly* decreasing in c_i, and

(ii) the optimal budget is *strictly* increasing in c_i.

Proof: Substituting the budget rule into the principal's objective function yields the following unconstrained maximization problem:

$$U_P = \frac{\pi \cdot h}{N \cdot M} \cdot \sum_{i=1}^{h} \left[R\bar{m}_i - \tfrac{1}{2}\bar{m}_i^{\,2} \right] - \frac{1}{N} \cdot \sum_{i=1}^{h} \left(c_i + \Delta \frac{\sum_{k=i+1}^{h} \bar{m}_k}{\bar{m}_i} \right).$$

(i) Maximizing the unconstrained optimization problem with respect to $\bar{m}_j$ yields

$0 = \frac{\pi \cdot h}{MN} \cdot \left(R - \bar{m}_j \right) + \frac{\Delta}{N} \left(\frac{\sum_{k=j+1}^{h} \bar{m}_k}{\bar{m}_j^{\,2}} - \sum_{i=1}^{j-1} \frac{1}{\bar{m}_i} \right)$. As the first-order condition has to hold for all $\bar{m}_j$, we can set $\partial U_P / \partial \bar{m}_j = \partial U_P / \partial \bar{m}_{j+1}$. Rearranging terms yields

$\bar{m}_j - \bar{m}_{j+1} = \frac{\bar{m}_{j+1} + \bar{m}_j}{\bar{m}_j^2} \left(\frac{\pi \cdot h}{\Delta \cdot M} + \frac{\left(\bar{m}_j + \bar{m}_{j+1} \right)}{\left(\bar{m}_j \bar{m}_{j+1} \right)^2} \sum_{k=j+2}^{h} \bar{m}_k \right)^{-1}$. The right-hand side is clearly strictly positive.

(ii) Substituting $\bar{m}_i > \bar{m}_{i+1}$ in the optimal budget from Corollary 2 establishes the claim. ■

Proof of Proposition 4:

(i) Maximizing the unconstrained optimization problem with respect to $\bar{m}_j$ yields

$0 = \frac{\pi \cdot h}{M} \cdot \left(R - \bar{m}_j \right) + \Delta \left(\frac{\sum_{k=j+1}^{h} \bar{m}_k}{\bar{m}_j^{\,2}} - \sum_{i=1}^{j-1} \frac{1}{\bar{m}_i} \right)$. Setting $j = 1$ yields

$0 = \frac{\pi \cdot h}{M} \cdot \left(R - \overline{m}_1 \right) + \Delta \frac{\sum_{k=2}^{h} \overline{m}_k}{\overline{m}_1^{\,2}}$ which establishes that $\overline{m}_1 > R$.

(ii) Setting $j = h$ in the above first order condition for $\overline{m}_j$ yields

$\overline{m}_h = R - \frac{\Delta}{\pi \cdot h} \sum_{i=1}^{h-1} \frac{M}{\overline{m}_i} < R$. ■

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