With a Grain of Salt: Uncertain Relevance of External Information and Firm Disclosures*

Jonathan Libgober[†] Beatrice Michaeli[‡] Elyashiv Wiedman[§]

May 15, 2023

*We thank Yaakov Amihud, Daniel Andrei, Snehal Banerjee, Sugato Bhattacharyya, Edwige Cheynel, Jesse Davis, Eti Einhorn, Felix Feng, Sivan Frenkel, Ilan Guttman, Valentin Haddad, Mirko Heinle, Bernard Herskovic, Navin Kartik, Ilan Kremer, Bart Lipman, Dan Luo, Nadya Malenko, Jordan Martel, Assaf Romm, Jacob Sagi, Ulrich Schäffer, Avanidhar Subrahmanyam, Tsahi Versano, Ran Weksler, Alex Zentefis as well as participants at 2023 Spring Meeting of the Finance Theory Group, 2020 Tel Aviv University Conference, 2020 Early Insights Conference, 2020 Accounting and Economics Society, 2019 Junior Theory Conference, Columbia Business School, Hebrew University of Jerusalem, Texas A&M, and Reichman University for helpful comments and suggestions on this and previous versions of this paper. Beatrice Michaeli acknowledges financial support from Laurence and Lori Fink Center for Finance at University of California in Los Angeles.

[†]Jonathan Libgober is at the Department of Economics, USC Dornsife College of Letters, Arts and Sciences, and can be reached at libgober@usc.edu.

[‡]Beatrice Michaeli is at the Anderson School of Management, University of California in Los Angeles, and can be reached at beatrice.michaeli@anderson.ucla.edu.

[§]Elyashiv Wiedman is at the School of Business Administration, Hebrew University of Jerusalem, and can be reached at elyashiv.wiedman@mail.huji.ac.il.

With a Grain of Salt: Uncertain Relevance of External Information and Firm Disclosures

Abstract: We examine how uncertainty about the relevance of third-party news ("external signal") influences investor beliefs, market prices and corporate disclosures. Despite assuming independence between signals' relevance and firms' information endowment, we find that favorable external signals are taken "with a grain of salt" in equilibrium—more precisely, perceived as less likely relevant—which reinforces investor beliefs that the firms are endowed with unfavorable information. As a result, more favorable external signals could paradoxically lead to lower market valuation. In line with mounting empirical evidence, we predict asymmetric price reactions to external news and price declines following firm disclosures.

Keywords: uncertain relevance, voluntary disclosure, asymmetric price reaction, price non-monotonicity

JEL: D82, D83, G12

1 Introduction

Stock markets rely on information from a variety of sources. These include disclosures from companies as well as news from outside sources such as analyst forecasts, peer reports, media broadcasts, and announcements of changes in macroeconomic factors, fiscal policies or regulations. One reason why external news might matter is if it conveys valuable insights about businesses. However, whether a given piece of news sheds light on firm value is not always clear-cut. For instance, outside news might reflect broad industry trends or suggest economic consequences for firms. However, such news might instead be the result of isolated and temporary disruptions and shocks, originate from an unreliable source or not come to fruition (e.g., if the policy is not approved or its specifics are substantially altered). In the former case, the market should update beliefs based on the news, but in the latter case it should not. The bottom line is that third-party information needs to be taken "with a grain of salt" if its *relevance (or reliability) is uncertain*. But how does uncertainty over the relevance of external news impact the information that companies themselves make public? And what are the implications for asset pricing?

This paper provides answers to these questions in the context of a dynamic disclosure model. We study the implications of uncertainty about relevance of external news and elucidate that it can influence investor beliefs, equilibrium disclosure behavior and market prices in subtle ways. Several of our results are in stark contrast to prior work set in disclosure settings that does not provide as central a role for belief updating about relevance. Among these results, most notable are our observations that beliefs are not constant (rather even discontinuous), and prices can be non-monotonic (and react asymmetrically), in the positivity of external news. Our contribution is to clarify why such phenomena naturally occur when markets anticipate the possibility that a given piece of news might not be relevant.

Our model features a manager ("she") who may be endowed with private information about firm value and, if informed, can voluntarily and truthfully disclose it to investors. The manager's objective is to maximize her firm's market price. The investors evaluate the firm after considering all publicly available information, which consists of the manager's disclosure and the information from an external source ("signal") with uncertain and unobservable relevance (alternatively, uncertain credibility or reliability of the source). For simplicity, we assume that if the signal is relevant, then it is equal to the firm value, and if it is irrelevant it represents an independent draw from the same distribution. This "truth-or-noise" signal structure allows for the uncertainty over the relevance to have the greatest impact possible, allowing us to study its implications most clearly, but is not critical for our main results. We take the arrival of the signal to be certain and its timing fixed—e.g., announcements of microeconomic factors and peers' financial statements have fixed release dates—but allow the manager to disclose at a cost either early (before the signal) or late (after the signal).

While the information endowment of the manager and the relevance of the signal are drawn independently, the investor beliefs about them are *endogenously intertwined* since both are updated jointly following nondisclosure. In principle, one may be concerned that this endogeneity creates scope for indeterminacy; for instance, if managers would disclose following a given signal when the market believes it to be relevant, but not when not. Nevertheless, we first show that *market inferences* about relevance are restricted in a particular way. Specifically, a signal that is relatively unfavorable must strictly strengthen joint beliefs that the signal is relevant and that the nondisclosing manager is informed. A relatively favorable signal has the opposite effect; such signal is unlikely to accurately describe the value of a firm run by an informed nondisclosing manager. This pattern is driven by the manager's (endogenous) strategic disclosure behavior and the information that can be inferred from it. In an equilibrium where relatively good news is disclosed and relatively bad news is withheld, the investors understand that an informed manager who did not disclose must have observed sufficiently bad news. Thus, if the external signal is relatively unfavorable, the investors perceive it as more likely relevant, strengthening their beliefs that the manager is informed. Conversely, investors cannot rationally infer that a favorable signal is relevant and that the silent manager is informed.

The shift in investor beliefs about relevance has two implications: First, the market price can be discontinuous and even *non-monotonic* in the signal so that relatively more favorable external news may paradoxically lead to lower market prices, since this news is viewed as less likely relevant. Second, the price is more sensitive to external news that is less favorable, since it is perceived as more likely relevant. This finding is consistent with mounting empirical evidence on asymmetric asset price reactions to external news in various settings (e.g., Aggarwal and Schirm 1998; Goldberg and Grisse 2013; Blot, Hubert and Labondance 2020; Capkun, Lou, Otto, and Wang 2022; Xu and You 2023).

The above-described property allows us to further show that managers with higher values are more eager to disclose, even though this need not hold under arbitrary (non-equilibrium) conjectures of disclosure strategies. This observation enables the derivation of existence and uniqueness of threshold equilibria (as is typical in disclosure models). To highlight the intuition behind these issues, we first focus on the case where disclosure can only be made late, after the external signal (e.g., because it is prohibitively costly to advance the disclosure or because the firm's conference call is scheduled after the release of macroeconomic news or a peer's financial report). We predict that relatively more favorable external news discourages the manager from disclosing, whereas unfavorable news encourages her—a prediction consistent with empirical evidence that a firm's likelihood to disclose information is influenced by the positivity of the information revealed by peers (Capkun, Lou, Otto, and Wang 2022). Our observation has a stark theoretical implication, namely that a *single* realization of the external signal splits the firm value into two regions: In the lower region, external news is relatively unfavorable and a higher likelihood of its relevance encourages disclosure. The opposite is true in the higher region where external news is relatively more favorable and a higher relevance likelihood discourages disclosure. Within each region, updating behavior is smooth and monotone; updating over relevance simply generates a kink in the market price at the common endpoint of the regions.

Different intuition emerges for the case where disclosure can only precede the external signal (e.g., because it is prohibitively costly to delay or because the firm's conference call is scheduled prior to the release of macroeconomic news or a peer's financial report). Since an informed manager decides whether to reveal her information before the signal, she considers the expected (future) market price, conditional on the value she observed. Even though her expectation of the nondisclosure price is non-monotonic and even discontinuous in the firm value, we still obtain a uniqueness result due to a key property: the price in this case exhibits a discontinuous jump *downwards*, and its expectation is below the firm values after the jump. We also find that the expected nondisclosure price exceeds low firm values in the region before the jump. In this region, the manager benefits from the uncertainty about relevance and relies on the external source to reveal the same information that she observed. Overall, we find that the presence of external sources discourages early corporate disclosures and this effect is exacerbated when the signal is more likely relevant.

Returning to the main dynamic disclosure setting, we note that if disclosure is cost-free, the manager at least weakly prefers to delay disclosure because external news might be favorable and there is no cost associated with waiting. This is also the case when advancing disclosure is more costly than delaying it. However, if advancing is relatively cheaper managers observing highly favorable information prefer to disclose early and avoid the (higher) cost of delaying. Thus, we predict that disclosures of favorable information occur before the arrival of external news and disclosures of less favorable information after that. Prior to concluding the paper, we incorporate frequent adjustment of market prices and find that it may encourage corporate disclosures. The predictions of this extension may offer an explanation of a finding in Sletten (2012) where a firm disclosing in response to a peer's restatement faces a decrease in its stock price.

Our paper is related to the literature on voluntary disclosure, initiated by Grossman (1981) and Milgrom (1981), and especially to the work about the impact of external news on disclosure when the market is uncertain whether firms have private information.[1] Several

[1] Uncertain information endowment is one of the frictions that prior literature finds to prevent information unraveling (Dye 1985). In the absence of any known friction, Einhorn (2018) finds that unraveling may also

papers (Frenkel, Guttman, and Kremer 2020; Dye and Sridhar 1995) consider settings where the arrival of external news depends on the firm's information endowment. In these studies the market updates beliefs about the manager's information endowment because of the assumed correlation—without it, there is no effect on beliefs. In contrast, we posit that the arrival of external news is uncorrelated with the manager's endowment but the news' relevance is uncertain. This substantially alters how investors update beliefs as well as how market prices react in our equilibrium, and opens the door for discontinuity and non-monotonicity. Furthermore, in the above-mentioned studies, the content of external news is either orthogonal to firm value with certainty (as in Dye and Sridhar 1995) or perfectly informative (as in Frenkel, Guttman and Kremer 2020). It is worth emphasizing that maintaining threshold equilibria with more general correlations between external news and firm value is not guaranteed: for some specifications, high types may prefer *not* to disclose (a point we discuss in Appendix B) or equilibria may only exist in mixed strategies (a possibility explored in recent work by Frenkel, Guttman, and Kremer 2023).

In several papers (e.g., Acharya, DeMarzo, and Kremer 2011; Menon 2020), the firm value and the external signal are imperfectly and positively correlated, with the correlation *known and fixed.* Thus, the arrival of the external signal affects the (conditional) distribution of the firm value, while investor beliefs about this correlation remain constant regardless of the external signal content. In contrast, we study how investors' uncertainty about the relevance of the external signal affects firms' information provision. We show that investor beliefs are endogenously intertwined, and therefore, they update in a *different manner across signal realizations* (despite assuming a constant prior correlation). We contribute to the literature by demonstrating how uncertainty over relevance introduces a "grain of salt" updating pattern that may lead to discontinuity of beliefs and non-monotonicity of the price as a function of the external signal.[2] To our knowledge, these predictions are a novel

be prevented when there are competing sources of information in the market.

[2]Past work on disclosure typically focuses on settings that generate prices that are smooth and monotonic in the external signal; See Appendix B for a more in-depth discussion of this point, including an argument for price monotonicity under log-concave joint distributions with constant correlation.

implication of the form of endogenous updating that we study.

Some of our results—that market prices can be discontinuous and non-monotonic and their reactions to news can be asymmetric—are reminiscent of those identified in several prior studies: however, the economic forces driving the results are substantially different. In Veronesi (1999) markets react asymmetrically because risk-averse investors hedge against changes in their uncertainty about the fundamental and overreact (underreact) to bad (good) news when times are good (bad). In Banerjee and Green (2015) asymmetric price reactions emerge because the traders have mean-variance preferences: bad news reduces the expected fundamental value and amplifies the negative risk effect, whereas good news increases it and counteracts the risk. In their model non-monotonicity occurs because for extremely good news the risk effect outweighs the mean effect thereby resulting in a price decrease.

In Bond, Goldstein and Prescott (2010), price discontinuity in the fundamental arises due to corrective actions that firms undertake after inferring information reflected in stock prices. In our model, the impact on prices is microfounded using a model of dynamic disclosure with external signals. This imposes constraints in equilibrium: for instance, ruling out the possibility of an upward jump in the price, whereas in their model this can occur depending on the impact of the intervention. In addition, the discontinuity we obtain is the case of *anticipatory* disclosure, and we do not obtain a discontinuity in the case where the manager's decision is corrective—i.e., when disclosure may be late. Lastly, in Genotte and Leland (1990) and Barlevy and Veronesi (2003) prices can be discontinuous due to exogenous insurance of investment portfolio or presence of uninformed investors who are willing to buy at relatively high prices and endogenously act as insurance.

2 Model and Benchmark

We extend the voluntary disclosure framework with uncertain information endowment by considering an additional source of public information with uncertain relevance.

Players and payoffs. The model entails a manager ("she") who runs a firm with terminal value $v \in [0, 1]$ and is a price-maximizer. A group of risk-neutral investors observes all publicly available information Ω, forms beliefs, and prices the firm at $P(\Omega) = \mathbb{E}[v|\Omega]$. In the main part of the paper we assume that the price is set once, after Ω is realized. In Section 5 we relax this assumption and allow investors to adjust the market price more frequently.

Information structure. The common prior belief is that v is drawn from a differentiable cumulative distribution function G (with a corresponding probability distribution function g) and has a prior expectation of $\mathbb{E}[v] = \mu$. The publicly available information Ω consists of an external signal (on occasion, just "signal") and the manager's voluntary disclosure:

1. *Exogenous signal.* There is an exogenous public signal s which may or may not be relevant. In particular, with probability $q \in (0, 1)$, the signal reflects the firm value, $s = v$; we say that the signal in this case is relevant ($\rho = R$). With probability $1 - q$, the signal is irrelevant ($\rho = N$) and $s = x$ where $x \in [0, 1]$ is drawn from the same probability distribution as the firm value.[3] As we show in Appendix C, the *truth-or-noise structure is not critical for our main results*, as long as relevant signals are sufficiently informative and irrelevant ones sufficiently uninformative. Signals with higher realizations are interpreted as "more favorable." Neither the manager nor the investors observe ρ. That is, the relevance of the signal is uncertain. (An alternative interpretation is that the reliability or credibility of the signal source is uncertain.)

2. *Manager's information endowment and voluntary disclosure.* With probability $p \in (0, 1)$, the manager is informed ($\kappa = I$) about the firm value, and otherwise is uninformed ($\kappa = U$). We assume that κ is independent of v and ρ and unknown to investors. An uninformed manager cannot credibly communicate the lack of information and has no choice but remain silent ($d = \varnothing$). An informed manager can voluntarily disclose the firm value ($d = v$) to investors before the arrival of the external signal (which we refer

[3] The support of an irrelevant external signal is the same as that of a relevant one—if this were not the case, the investors could learn whether the external signal is relevant for some signal values.

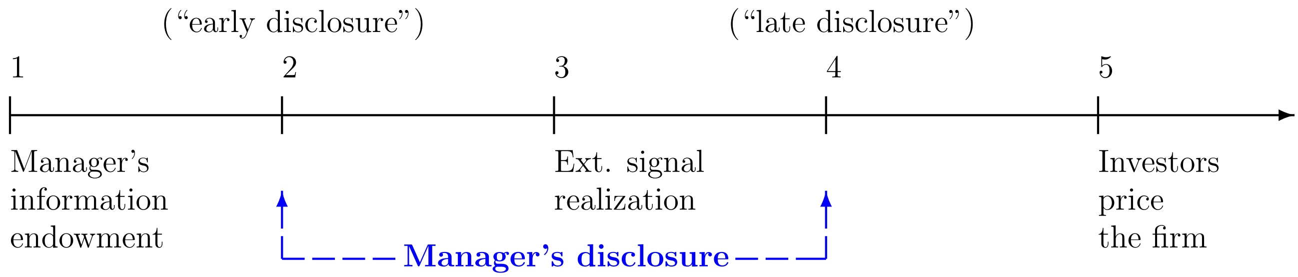

Figure 1: Timeline of events

to as "early disclosure") at cost $c^E \geq 0$ or after (which we refer to as "late disclosure") at cost $c^L \geq 0$. Following the voluntary disclosure literature, any disclosed value is verifiable. We further assume that in equilibrium, a manager who is indifferent between disclosure decisions chooses to remain silent.[4] We describe the outcomes without assumptions on how indifference is broken in Section 4.2. In this section we also show that, except for the indifference point, the manager always has strict preferences. Thus, *while we allow for mixed strategies in our analysis, these do not arise in equilibrium.*

Timeline. The timeline of events is illustrated in Figure 1. At date 1, the manager observes the firm value v with probability p. At date 2, the manager decides whether to disclose the observed value (early disclosure). At date 3, the external signal s is realized. At date 4, if the manager has not disclosed earlier, she can disclose now (late disclosure). At date 5, the investors price the firm.

Benchmark. As a benchmark (superscript "B"), suppose there is no signal (as in Dye 1985; Jung and Kwon 1988) or the signal is irrelevant with certainty ($q = 0$) and there are no disclosure costs ($c^E, c^L = 0$). The price following disclosure is $P(v) = v$. The price following

[4] Assuming a particular tie-breaking rule is common in the literature as a way to avoid equilibrium multiplicity. Our results characterize all equilibria, even without this assumption.

nondisclosure is

$$P(\varnothing) \quad = \quad \Pr(I|\varnothing) \cdot \mathbb{E}[v|v \leq v^B] + \Pr(U|\varnothing) \cdot \mu \tag{1}$$

when the investors conjecture that an informed manager discloses all values above a threshold v^B.[5] In equilibrium, when the manager observes $v = v^B$, she is indifferent between disclosing and withholding her information. That is, $P(\varnothing) = P(v) = v^B$.

Lemma 1 (Dye 1985; Jung and Kwon 1988)**.** *When* $q, c^E, c^L = 0$, *there exists a unique threshold* $v^B \in (0, \mu)$, *such that the manager discloses if* $v > v^B$ *and withholds otherwise. The threshold* v^B *is decreasing in the probability of information endowment,* p.

The key insight in Dye (1985) is that informed managers can pretend to be uninformed because the market is uncertain about their information endowment. In this paper, we show that exogenous signals with uncertain relevance affect the ability of managers to pretend to be uninformed in two ways. First, because these signals may provide information about firm value, they directly affect the market price and the managers' disclosure decisions. Second, the signals also influence the investor beliefs regarding the managers' information endowment and the signal relevance. As a result, the signals indirectly affect the market price and corporate disclosures. This second channel drives many of our observations, such as beliefs' discontinuity and prices' non-monotonicity.

3 Investor Beliefs and Market Prices

We begin the analysis by illustrating several key features of investor beliefs and market prices that arise in our setting. To do so, for now we assume that the equilibrium disclosure follows a threshold rule—a result that we formally prove in Section 4. Similar to the benchmark

[5] Here, $\Pr(I|\varnothing) = \frac{p \cdot G(v^B)}{1-p+p \cdot G(v^B)}$ and $\Pr(U|\varnothing) = 1 - \Pr(I|\varnothing)$. Note that these conditional probabilities also depend on the disclosure threshold—but we suppress it to avoid clutter.

case, the market price when the manager discloses is determined only by the disclosed value,

$$P(s, v) = \mathbb{E}[v|v, s] = v. \tag{2}$$

The investors disregard the external signal because the manager observes the firm value precisely and her disclosure is truthful.

When the manager remains silent, the price depends on the investor beliefs about the manager's information endowment and the signal relevance. Specifically, the investors consider four possible events: (i) the manager is uninformed and the signal is irrelevant; (ii) the manager is uninformed and the signal is relevant; (iii) the manager is informed and the signal is irrelevant; (iv) the manager is informed and the signal is relevant. Therefore the market price can be expressed as:

$$\begin{aligned} P(s, \varnothing) = \mathbb{E}[v|s, \varnothing] \quad = \quad & \Pr(U, N|s, \varnothing) \cdot \mathbb{E}[v|U, N, s, \varnothing] + \Pr(U, R|s, \varnothing) \cdot \mathbb{E}[v|U, R, s, \varnothing] \\ & + \Pr(I, N|s, \varnothing) \cdot \mathbb{E}[v|I, N, s, \varnothing] + \Pr(I, R|s, \varnothing) \cdot \mathbb{E}[v|I, R, s, \varnothing]. \end{aligned} \tag{3}$$

To simplify (3), consider the case where the investors believe that the signal is irrelevant and the manager is uninformed. Then, because there is nothing to be learned from the signal and the manager's silence, the market expectation about firm value is simply the prior, $\mathbb{E}[v|U, N, s, \varnothing] = \mathbb{E}[v] = \mu$. If investors believe the signal is relevant, their expectation of the firm value is simply the signal, $\mathbb{E}[v|\kappa, R, s, \varnothing] = s$, regardless of whether they believe the manager is informed ($\kappa = I$) or not ($\kappa = U$). This is because a relevant signal perfectly reflects the firm value. The last case (when investors believe the manager is informed and the signal is irrelevant) introduces sensitivity of the equilibrium price function to the equilibrium disclosure threshold. Note that, in this case, there is nothing to be learned from the signal, and so the investors disregard it. The nondisclosure decision, however, indicates that the manager prefers to withhold the observed value. As we formally show in Section 4, the manager withholds values that are lower than some threshold $\widehat{v} \in [0, 1]$

and discloses otherwise.[6] Thus the market expectation about the firm value in this case is $\mathbb{E}[v|I,N,s,\varnothing] = \mathbb{E}[v|v \le \widehat{v}]$. We can simplify the nondisclosure market price in (3),

$$\begin{aligned} P(s,\varnothing) &= \Pr(U,N|s,\varnothing)\cdot\mu + \Pr(U,R|s,\varnothing)\cdot s \\ &\quad + \Pr(I,N|s,\varnothing)\cdot\mathbb{E}[v|v\le\widehat{v}] + \Pr(I,R|s,\varnothing)\cdot s. \end{aligned} \tag{4}$$

Our next results describe the probabilities the market assigns to each possibility ρ and κ, and illustrate how the beliefs of the investors about the occurrence of these events are affected by the observed signal in a nontrivial way (despite the fact that they are initially independent).

Lemma 2. *Let $\gamma(s,\widehat{v}) \equiv q(1-\mathbb{1}_{s>\widehat{v}}\cdot p) + (1-q)\big(1-p+pG(\widehat{v})\big)$. For a given threshold $\widehat{v}$, the investor beliefs conditional on $d=\varnothing$ and s are given by*

$$\begin{aligned} \Pr(I,R|s,\varnothing) &= \frac{qp\cdot\mathbb{1}_{s\le\widehat{v}}}{\gamma(s,\widehat{v})}, \qquad \Pr(I,N|s,\varnothing) = \frac{(1-q)pG(\widehat{v})}{\gamma(s,\widehat{v})}, \\ \Pr(U,R|s,\varnothing) &= \frac{q(1-p)}{\gamma(s,\widehat{v})}, \qquad \Pr(U,N|s,\varnothing) = \frac{(1-q)(1-p)}{\gamma(s,\widehat{v})}. \end{aligned}$$

Theorem 1. *For a given threshold $\widehat{v}$ the joint investor beliefs about ρ and κ depend on the external signal and are discontinuous. In particular,*

$$\begin{aligned} \lim_{s\to\widehat{v}^-}\Pr(\kappa,\rho|s,\varnothing) &> \Pr(\kappa|\varnothing)\Pr(\rho) > \lim_{s\to\widehat{v}^+}\Pr(\kappa,\rho|s,\varnothing) \text{ if } \rho=R \text{ and } \kappa=I, \\ \lim_{s\to\widehat{v}^-}\Pr(\kappa,\rho|s,\varnothing) &< \Pr(\kappa|\varnothing)\Pr(\rho) < \lim_{s\to\widehat{v}^+}\Pr(\kappa,\rho|s,\varnothing) \text{ otherwise.} \end{aligned}$$

At the heart of our results, especially the price non-monotonicity that our model exhibits, is how the market makes inferences about the signal's relevance based on the manager's disclosure behavior. The intuition behind the downward jump in the joint beliefs about the event $(\rho=R,\kappa=I)$ is particularly insightful. In an equilibrium with disclosure threshold $\widehat{v}$, the investors understand that an informed manager who did not disclose must have observed

[6] The threshold may depend on the signal, but we repress this dependence for now.

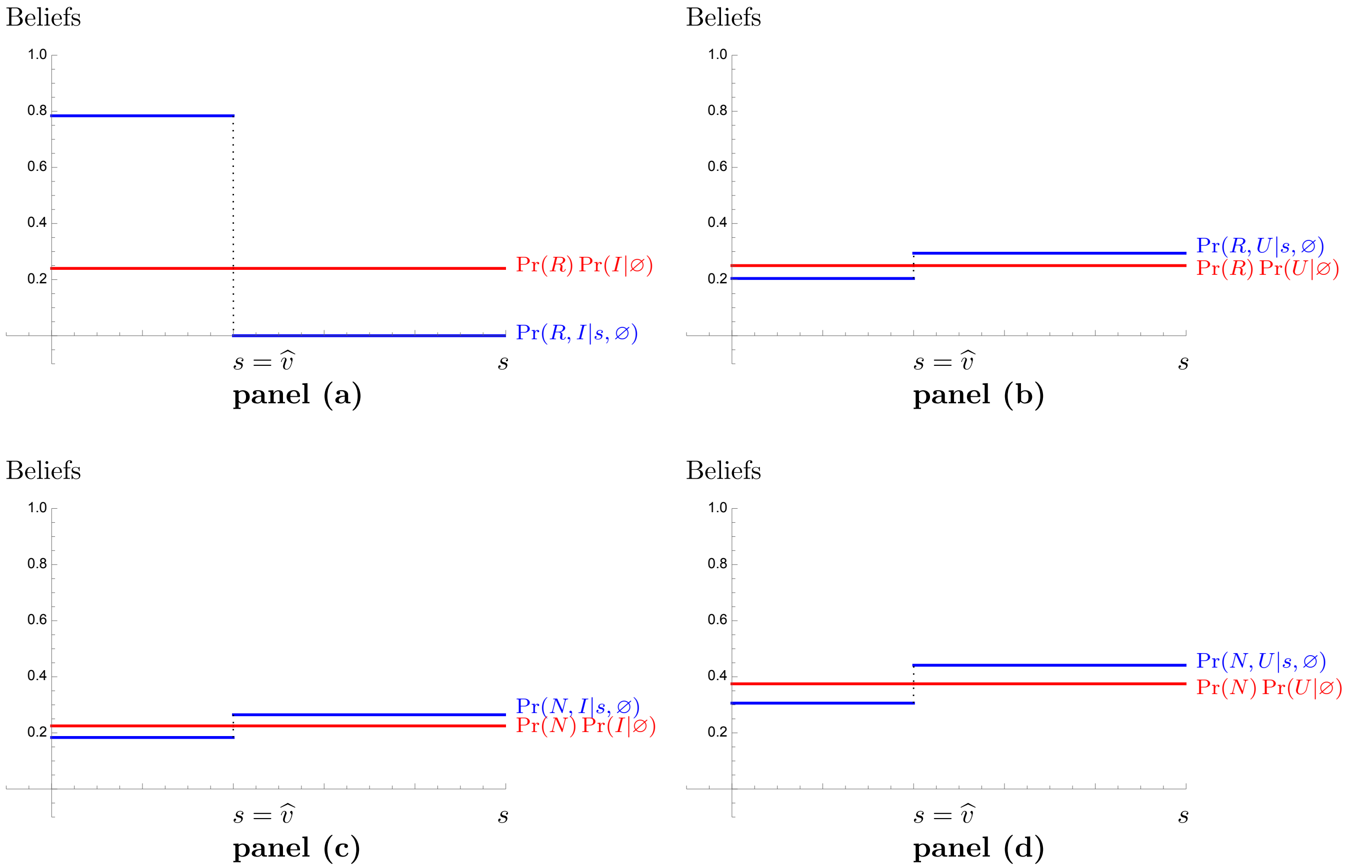

Figure 2: Joint investor beliefs about ρ and κ at $s = \widehat{v}$
Numerical example with uniform distribution, $\Pr(I) = p = 0.6$, $\Pr(R) = q = 0.4$.
Under these parameter values, $\widehat{v} = 0.4$, $\Pr(U|\varnothing) = 0.625$ and $\Pr(I|\varnothing) = 0.375$.

$v \leq \widehat{v}$.[7] Thus, if the external signal exceeds the disclosure threshold ($s > \widehat{v}$), it will be rationally inconsistent for the investors to believe that the signal is relevant and that the manager is informed at the same time. Thus, they infer that $\Pr(I, R|s > \widehat{v}, \varnothing) = 0 < \Pr(I|\varnothing)\Pr(R)$. In contrast, investors perceive a signal below the disclosure threshold ($s \leq \widehat{v}$) as more likely to be relevant, which strengthens their joint beliefs that the nondisclosing manager is also informed—in this case, $\Pr(I, R|s \leq \widehat{v}, \varnothing) > \Pr(I|\varnothing)\Pr(R)$. This illustrates how beliefs about relevance are non-constant in the value of the signal, at least holding the disclosure threshold fixed.[8] It is illustrated in panel (a) of Figure 2. The rest of the joint

[7] We formally prove the existence of unique threshold equilibrium in Theorem 4.

[8] It can be shown that $\lim_{s\to\widehat{v}^-}\Pr(R|s,\varnothing) > \Pr(R) > \lim_{s\to\widehat{v}^+}\Pr(R|s,\varnothing)$ and $\lim_{s\to\widehat{v}^-}\Pr(I|s,\varnothing) >$

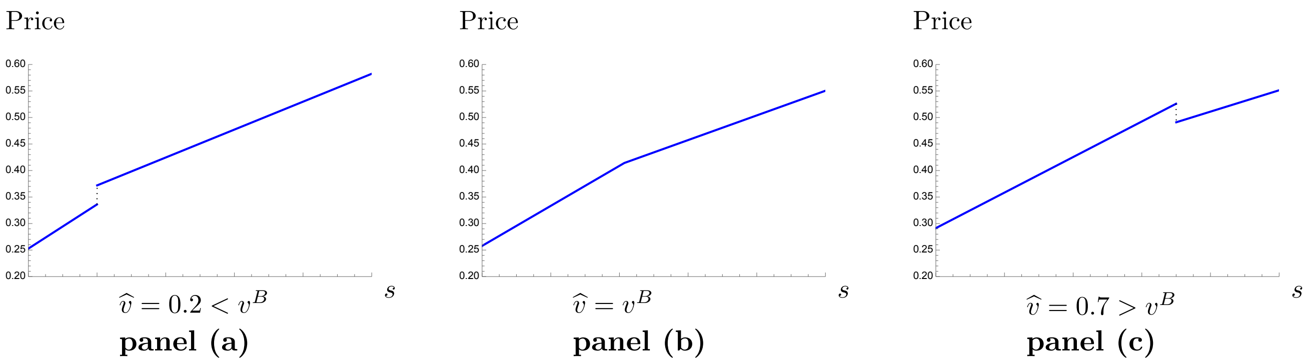

Figure 3: Nondisclosure price for given $\widehat{v}$ as a function of the external signal s
Numerical example with uniform distribution, $p = 0.5$, $q = 0.3$. Here, $v^B = 0.42$.

beliefs are illustarted in panels (b), (c) and (d). All of these beliefs are discontinuous with an upward jump at $s = \widehat{v}$.

The preceding discussion implies that unfavorable s strengthens the beliefs of investors that the external information is relevant and that the nondisclosing manager is informed. Conversely, favorable s weakens the investor beliefs: such information is less likely to accurately describe the value of a firm run by an informed, nondisclosing manager.

Putting our observations together, it is straightforward to provide a formal description of the market price following manager's silence $d = \varnothing$ and external signal s.

Lemma 3. *Let $\gamma(s, \widehat{v}) \equiv q(1 - \mathbb{1}_{s>\widehat{v}} \cdot p) + (1 - q)\big(1 - p + pG(\widehat{v})\big)$. For a given disclosure threshold $\widehat{v}$, the nondisclosure price is:*

$$P(s, \varnothing) = \frac{q(1 - \mathbb{1}_{s>\widehat{v}} \cdot p)s + (1 - q)((1 - p)\mu + pG(\widehat{v})\mathbb{E}[v|v \leq \widehat{v}])}{\gamma(s, v)}. \tag{5}$$

These observations drive the following pair of results, which clarify how the market's inference about relevance substantively influences the features of the market price:

Theorem 2. *For given threshold $\widehat{v}$ the nondisclosure price is discontinuous in the external*

$\Pr(I|\varnothing) > \lim_{s\to\widehat{v}^+} \Pr(I|s, \varnothing)$.

signal if $\widehat{v} \neq v^B$. *In particular,* $\lim_{s\to\widehat{v}^-} P(s,\varnothing) \gtrless \lim_{s\to\widehat{v}^+} P(s,\varnothing)$ *if* $\widehat{v} \gtrless v^B$ *but* $\lim_{s\to\widehat{v}^-} P(s,\varnothing) = \lim_{s\to\widehat{v}^+} P(s,\varnothing)$ *if* $\widehat{v} = v^B$.

Theorem 3. *For given threshold* $\widehat{v}$ *the nondisclosure market price is more sensitive to sufficiently unfavorable external news,* $\frac{\partial P(s,\varnothing|s\leq\widehat{v})}{\partial s} > \frac{\partial P(s,\varnothing|s>\widehat{v})}{\partial s} > 0$.

Three observations are worth emphasizing. First, because investors are uncertain about the relevance of the signal, they never price the firm at s (except for a knife-edge case). Second, the investors' reaction is more sensitive to sufficiently unfavorable signals ($s < \widehat{v}$) than to sufficiently favorable ones ($s > \widehat{v}$).[9] In our setting, the reason is that unfavorable signals are perceived to be more likely relevant than favorable signals (Theorem 1).

Third, *for a given threshold*, the nondisclosure price might be discontinuous with an upward (Figure 3 panel a) or downward (Figure 3 panel c) jump. As we show in Theorem 6, the jump *can never be upwards in equilibrium* (i.e., panel a cannot happen).[10] With a downward jump, favorable external information does not necessarily increase stock prices, i.e., the nondisclosure price is non-monotonic in the external news. When the manager shares the firm value with investors, the market price does not depend on the external signal because the manager's information is accurate and her disclosure is truthful. When the manager remains silent, the market price is increasing in the signal but (except in the knife-edge case of $\widehat{v} = v^B$) there is a discontinuity at $s = \widehat{v}$ such that $\lim_{s\to\widehat{v}^-} P(s,\varnothing) \neq \lim_{s\to\widehat{v}^+} P(s,\varnothing)$. This is because the investor beliefs are shifted when the signal reaches the disclosure threshold $\widehat{v}$.[11] As a result, in the neighborhood of the disclosure threshold, better external news leads

[9] It has been documented that asset prices may react asymmetrically to external news such as announcements of US trade balances (Aggarwal and Schirm 1998), macroeconomic news (Goldberg and Grisse 2013); Federal Open Market Committee decisions (Blot, Hubert and Labondance 2020); peers' clinical trial results (Capkun, Lou, Otto, and Wang 2022); and Initial Jobless Claims (Xu and You 2023). While it is impossible to determine whether these asymmetric reactions are related to the economic forces in our model, we note that our results are consistent with the empirically observed phenomena.

[10] For some intuition, note that if the price were to jump upward at a value where the manager were indifferent between disclosure decisions, then those to the right of this value would find it optimal to not disclose, just as the managers to the left of it, eliminating the possibility of a jump.

[11] In Appendix D, we mention that the non-monotonic updating patterns remain even when the truth-or-noise structure is relaxed, provided relevant signals are still viewed as sufficiently informative (and irrelevant signals sufficiently uninformative). We also mention that smooth noise will "smooth out" the price function, but will nevertheless still involve a steep drop at $\widehat{v}$, qualitatively similar to the discontinuous case.

to a lower nondisclosure price if $\widehat{v} > v^B$ (as is the case of panel c).

The potential for discontinuity and non-monotonicity of the market price in s arises because of the rational way in which the market makes inferences about the signal relevance and the manager's information endowment (Lemma 2 and Theorem 1). To appreciate the importance of this updating, consider naive investors who, regardless of the external signal value and the manager silence, fix their joint beliefs at $\Pr(\rho)\Pr(\kappa|\varnothing)$. Then, the nondisclosure price in (4) would simply be given by $\Pr(R)\Pr(U|\varnothing)\cdot s + \Pr(R)\Pr(I|\varnothing)\cdot s + \Pr(N)\Pr(U|\varnothing)\cdot \mu + \Pr(N)\Pr(I|\varnothing)\cdot \mathbb{E}[v|v \leq \widehat{v}]$, which is continuous and monotonic in s.

Aside from being economically substantive, price discontinuity and non-monotonicity usually present technical difficulties with showing equilibrium existence (or fully characterizing equilibria). As we show in Section 4, despite the phenomenon described in Theorem 2, there exists a unique equilibrium in our game.

4 Disclosure Incentives

Throughout this section, the superscript "L" denotes late timing and the superscript "E" early. We first observe that the equilibrium disclosure follows a threshold rule in both early and late timing. In the latter case the disclosure threshold is a function of the signal.

Theorem 4. *Given arbitrary $c_E, c_L \geq 0$, there exists a threshold equilibrium of the game; that is, a number $v^E \in (v^B, 1]$ and an increasing, continuous function $v^L(s)$ such that the manager (i) discloses early if $v > v^E$ and (ii) discloses late if disclosure was not early and $v \in (v^L(s), v^E]$; otherwise, she remains silent.*

Theorem 5. *If $g(\cdot)$ is log-concave, there exists exactly one threshold v^E and increasing, continuous function $v^L(s)$ which form an equilibrium.*

Similar to prior studies, the simultaneous presence of disclosure costs and uncertain information endowment may result in multiplicity of equilibria. Our assumption that $g(\cdot)$ is from the log-concave family of distributions is a sufficient condition to rule out this possibility (e.g., Bagnoli and Bergstrom 2005; Kartik, Lee and Suen 2019). While equilibrium in threshold strategies is common in settings with dynamic disclosure (such as Acharya, DeMarzo and Kremer 2011; Kremer, Schreiber and Skrzypacz 2021), we note that our proof technique is different since it requires special care in tracking that the inferences about relevance are well-behaved. Importantly, we also show the following:

Theorem 6. *For threshold equilibria (as defined in Theorem 4), the nondisclosure price is either discontinuous in* s *with a downward jump or continuous, i.e.,* $\lim_{s \to \min\{v^E, v^L(s)\}^-} P(s,v) \geq \lim_{s \to \min\{v^E, v^L(s)\}^+} P(s,v)$. *In either case, the sensitivity of the market price to the external signal in the equilibrium is not constant.*

The reasons we obtain a possibly downward discontinuous jump are clarified below; for now, we mention that this emerges due to a jump in the belief of the signal's relevance in the presence of early disclosure. We explain this more thoroughly in Section 4.2.

The proofs and intuitions underlying the above results are quite involved. We find it more illuminating to first study and clarify the intuition in the case where disclosure is either only early or only late. This will allow us to present not only the practical implications of our model on disclosure, but also the key properties of the solution that allow us to prove these theorems more generally. Note that in our dynamic disclosure case, the *option value* from delay is non-constant in the manager's type, and itself is a function of the market's equilibrium inference. This creates endogenous feedback between the disclosure decision and the relative benefit from nondisclosure. While the proof shows that this does not prevent the existence of threshold equilibria in our setting, this complexity has been noted in past work (e.g., Frenkel, Guttman, and Kremer 2020); thus, one of our key technical innovations is to articulate why our "truth-or-noise" information environment can circumvent these issues.

4.1 Late Disclosure

We begin with a late disclosure scenario in which the manager observes the external signal and then decides whether to disclose her information. This could happen for example when

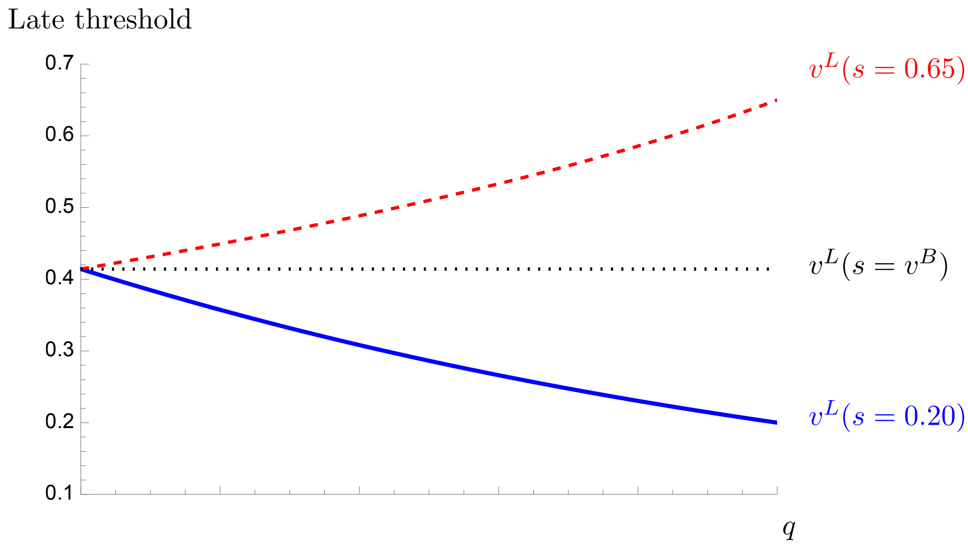

Figure 4: Late disclosure threshold as a function of q
Numerical example with uniform distribution, $c^E = 0$, $c^L > 1$, $p = 0.5$. Here, $v^B = 0.42$.

macroeconomic news or a peer's financial report are released before the manager's conference call.[12] Within our model framework we capture this scenario by assuming $c^E > 1$ and $c^L = 0$: while Theorem 4 shows that the equilibrium is characterized by two thresholds, here the prohibitively high c^E implies that $v^E = 1$ and hence early disclosure is shut down.

An informed manager can respond by disclosing (late) the observed value if the disclosure price in (2) exceeds the nondisclosure one in Lemma 3.

Proposition 1. *Suppose $c^L = 0$ and $c^E > 1$. Then, the manager discloses if the value exceeds $v^L(s) < 1$. This threshold is increasing in q if $s > v^B$ but decreasing if $s < v^B$. Furthermore, if $s \lesseqgtr v^B$ then $v^L(s) \gtreqless s$ and $v^L(s) \lesseqgtr v^B$.*

Managers observing low v benefit from the investors' uncertainty about the relevance of s because: (i) if the signal is relevant and thus equals the low v, it will be given a low weight in the price formation and (ii) if the signal is not relevant it could be more favorable than the true value of the firm. Thus managers observing relatively low v prefer to remain silent. The opposite holds for managers observing high firm value—they do not benefit from the investors' uncertainty and prefer to respond by "correcting" external signals.

[12]The manager may also observe the firm value only after the arrival of external information. The results in Section 4.1 hold qualitatively under such alternative timeline.

To understand the role of the benchmark threshold v^B in our result, note that if the signal is relevant with certainty, $q = 1$, the nondisclosure price equals s. But if $q \in (0, 1)$, the market expectation is a convex combination of the price when s is relevant for sure ($\lim_{q \to 1} P(s, \varnothing) = s$) and when it is not relevant for sure ($\lim_{q \to 0} P(s, \varnothing) = P(\varnothing) = v^B$). This implies that when $s = v^B$, the non-disclosure price (and thereby the late disclosure threshold) is *independent* of the signal's perceived relevance, since it is equal to v^B no matter what this conjecture is. It turns out that $s = v^B$ is the only signal with this property. For any $s \neq v^B$ the effect of q is non-trivial as graphically illustrated in Figure 4. Intuitively, if the signal is sufficiently favorable, $s > v^B$, higher q implies that investors believe the signal is more likely relevant and put a higher weight on this more favorable signal, which strengthens the incentives of the manager to remain silent, i.e., $v^L(s)$ increases. The opposite is true when the signal is sufficiently unfavorable, $s < v^B$. Then, higher q means investors put a higher weight on this unfavorable signal which stimulates the manager to respond. That is, $v^L(s)$ decreases. Put differently, whether external news encourages or discourages disclosure depends on s. A manager facing a sufficiently unfavorable (favorable) signal is more (less) likely to disclose compared with the benchmark case, only a signal $s = v^B$ has no effect on the probability of disclosure. This is consistent with the empirical findings in Capkun, Lou, Otto, and Wang (2022) documenting that the likelihood of voluntary disclosure depends on the positivity of the news conveyed by a peer.

It seems intuitive that managers respond and "correct" unfavorable external signals. Our result, however, shows that this intuition is not always true. If the external signal is sufficiently low ($s < v^B$), the disclosure threshold exceeds it ($v^L(s) > s$); i.e., the manager withholds values that are *more favorable* than the ones revealed by the external signal. Conversely, if the external signal is sufficiently high ($s > v^B$), the disclosure threshold falls short of it ($v^L(s) < s$); i.e., the manager discloses values that are *less favorable* than the ones revealed by the external signal. This result, graphically illustrated in Figure 5, may at first seem perplexing: why would anyone withhold (relatively) favorable news but

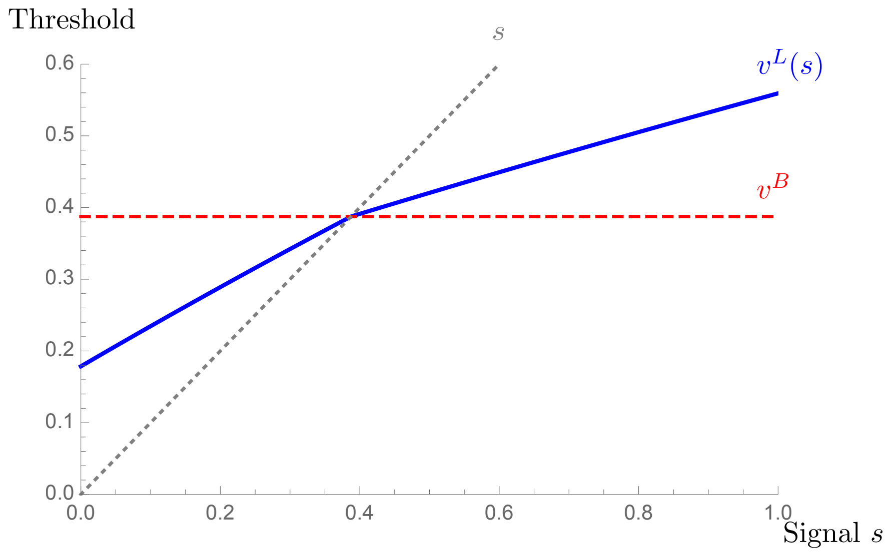

Figure 5: Late disclosure threshold as a function of s
Numerical example with uniform distribution, $c^E > 1$, $c^L = 0$, $p = 0.6$ and $q = 0.4$.

reveal (relatively) unfavorable news? The answer is simple: the manager discloses when the observed value exceeds the nondisclosure price. Because the latter is not identical to s (recall that investors are skeptical about the signal's relevance and under-react to it), remaining silent in the face of some unfavorable s and "speaking up" to correct some favorable external news may be beneficial. The preceding discussion highlights a general property of the equilibrium in our model (that arises even without the restrictions on disclosure costs): there is a unique "switch point" such that the threshold $v^L(s)$ is above s for low values of s and below s for high values.

Proposition 2. *Suppose $c^L = 0$ and $c^E > 1$. In the late disclosure equilibrium, the nondisclosure price is continuous in s, i.e., $\lim_{s \to v^L(s)^-} P(s, v) = \lim_{s \to v^L(s)^+} P(s, v)$. In addition, $v^L(s)$ is continuous in s with $1 > \frac{\partial}{\partial s} v^L(s|s \leq v^B) > \frac{\partial}{\partial s} v^L(s|s > v^B) > 0$.*

We find that $v^L(s)$ is increasing in the external signal and is more sensitive to unfavorable news (Figure 5). Furthermore, the equilibrium threshold is continuous in s even though the nondisclosure price differs for favorable and unfavorable external news. The intuition for this result can be found in Proposition 1. The exact signal realization that distinguishes between the two pricing options is $s = v^B$. If the external signal is below this value, it is also above the late disclosure threshold. And vice versa, if s is above this value it is below $v^L(s)$. At the critical value, it holds that $s = v^B = v^L(s)$ at which point the two pricing options coincide.

4.2 Early Disclosure

We now consider the alternative case of early disclosure where the manager decides whether to disclose her private information before the arrival of the external signal. This could happen when the manager has a scheduled conference call before announcement of macroeconomic news or release of a peer's financial statements. Within our model framework we capture this scenario by assuming $c^L > 1$ and $c^E = 0$.

A manager who is not endowed with information has no choice but to remain silent. An informed manager has to consider the market price following nondisclosure, $P(s, \varnothing)$. Because this price depends on a signal that is not yet available at the time of the early disclosure decision, the manager has to form an expectation about it, conditional on the observed v. Note that v carries information about the signal because the latter may, with some probability, accurately reflect firm value. From the manager's perspective at date 2, the future external signal will be either relevant or irrelevant. Thus a straightforward way to think about the expected price is to present it as

$$\mathbb{E}[P(s, \varnothing)|v] = \Pr(R|v) \cdot \mathbb{E}[P(s, \varnothing)|R, v] + \Pr(N|v) \cdot \mathbb{E}[P(s, \varnothing)|N, v].$$

Because the signal is not realized yet, the observed value v carries no information about the signal relevance—thus $\Pr(\rho|v) = \Pr(\rho)$, $\rho \in \{R, N\}$. The manager expects that a relevant signal equals the observed value v. Hence, with probability $\Pr(R) = q$, the expected nondisclosure price is $\mathbb{E}[P(s, \varnothing)|R, v] = P(s = v, \varnothing)$. An irrelevant signal equals x so that, with probability $\Pr(N) = 1 - q$, the expected price is $\mathbb{E}[P(s, \varnothing)|N, v] = \mathbb{E}[P(s = x, \varnothing)]$.

When deciding whether to disclose, an informed manager compares her expectation of the nondisclosure price with the price in case of disclosure. Because both prices depend on v, it is not immediately obvious that a threshold equilibrium exists. Theorem 4 shows that

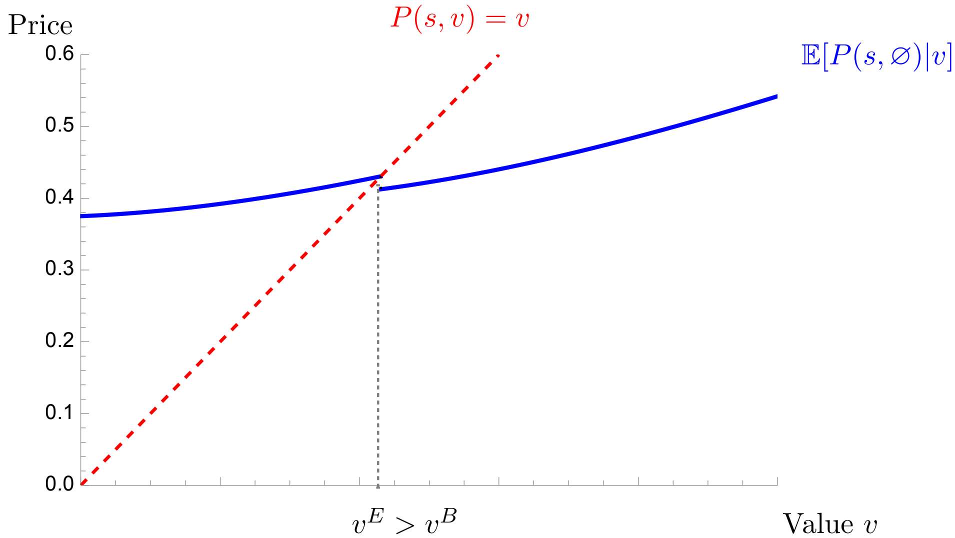

Figure 6: Market price and equilibrium (early) disclosure threshold
Numerical example with uniform distributions, $c^E = 0$, $c^L > 1$, $p = 0.6$ and $q = 0.4$.
The disclosure price is illustrated with the dashed red line, and the expected nondisclosure price with the solid blue line.

despite this endogeneity the usual increasing difference property holds, conditional on the relevance of the external signal.

The expected nondisclosure price faced by the manager is increasing in the observed v (via the expectation $\mathbb{E}[s|v]$), up until the point at which the market expects disclosure to become more beneficial than keeping silent—at this point, the price discontinuously jumps. Theorem 2 established that, for an *exogenous* threshold, this jump can be upward or downward. However, *in the equilibrium* of the early case, this jump *can only be downward*. Furthermore, $v^E > v^B$ as the result below establishes.

Proposition 3. *Suppose $c^E = 0$ and $c^L > 1$. In the early disclosure equilibrium, the nondisclosure price is discontinuous in s with a downward jump, i.e., $\lim_{s\to v^{E-}} P(s,\varnothing) > \lim_{s\to v^{E+}} P(s,\varnothing)$. Furthermore, early disclosure threshold v^E satisfies $v^E \in (v^B, \mu)$.*

The expected arrival of external news crowds out managerial disclosure compared with the baseline benchmark case. Managers who withhold under the benchmark continue to withhold in the presence of external information. Moreover, managers observing $v \in [v^B, v^E]$ also withhold their information—we predict that external information of uncertain relevance crowds out voluntary disclosure. Another way to interpret our result is to say that some managers are better off when their information is revealed by a potentially relevant external source rather than when they directly disclose it. This is driven by the investors' uncertainty about the relevance of external news and the manager's information endowment. Specifically, if investors believe that the external signal is relevant, they react the same way as they would have if the manager had disclosed the information. However, if they believe that the external source is irrelevant, the investors assign a higher likelihood that the manager is uninformed and hence put more weight on the prior expectation. Thus some managers observing values below μ benefit from relying on the external source to reveal those values.

For intuition behind the downward jump, it is instructive to consider the case where q is very large. Recall that, if $s = v^B$, the market price is independent of beliefs about relevance; thus, a manager with value v^B knows the price will simply be the price in Equation (1) if the signal is relevant (i.e., v^B itself). However, as $q \to 1$, the *expectation* of the price conditional on the signal being irrelevant approaches $\mu > v^B$. The property that an independently drawn external signal is more favorable than v^B turns out to hold for all q, allowing us to show that the only thresholds can be above v^B (and hence inducing downward jumps).

Further extending our observation that $v^E > v^B$, we confirm numerically for the uniform distribution that the early threshold is increasing in the prior probability that the signal is relevant (see Figure 7 for graphical illustration). Put differently, more informative external signals crowd out managerial disclosures more strongly.

Note that the manager's belief about the market perception of relevance is itself stochastic under early disclosure (unlike late disclosure). When the manager can only disclose early, the nondisclosure price is non-monotonic in the external signal in the sense that, even though this price is increasing in s there is a discontinuity at the threshold such that $\lim_{s \to v^{E-}} P(s, \varnothing) > \lim_{s \to v^{E-}} P(s, \varnothing)$ (by Theorem 2 and because $v^E > v^B$). In the neighborhood of the early disclosure threshold, the firm price may decrease after favorable external news. This non-monotonicity reflects the higher skepticism that favorable external signals are viewed with—that is, that they are taken "with a grain of salt"—as the fact that the manager did

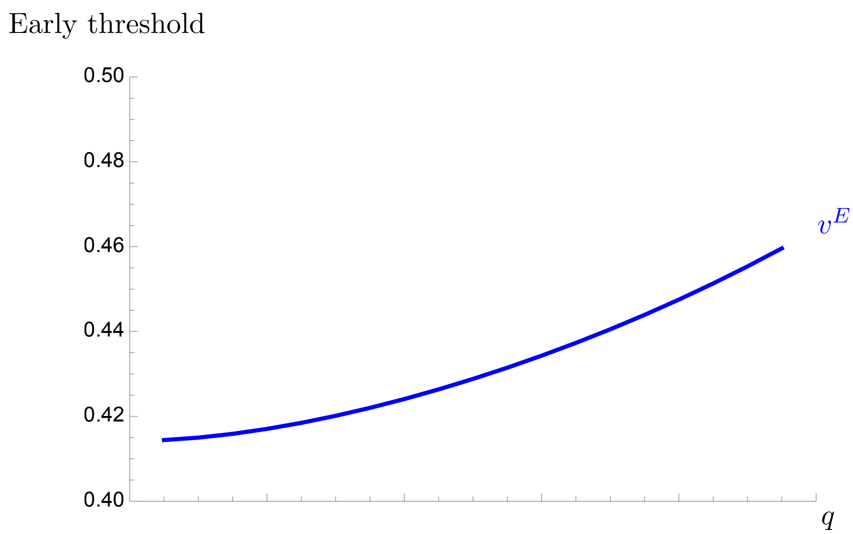

Figure 7: Early disclosure threshold as a function of q
Numerical example with uniform distribution, $c^E = 0$, $c^L > 1$, $p = 0.5$

not disclose means that the signals are more likely to reflect noise. This is consistent with stock price decreases after the release of good news from a peer firm (Sletten 20112).

We conclude with a comment on the assumption that an indifferent manager remains silent. This assumption plays no role in the proof of Theorem 4, but avoids the possibility of multiplicity in Theorem 5. Notice that in Figure 6, when the manager is indifferent, the expected price function is left continuous; left continuity emerges because the market assumes indifferent managers do not disclose (as per our solution concept), so that the same disclosure behavior emerges at v^E, as well as for all $v < v^E$. On the other hand, if the indifferent manager *discloses*, the expected price would be *right*-continuous, and only intersect the 45 degree line *after* "jumping downward." This would yield a different price function. More generally, varying the tiebreaking probability (mixing strategy) would result in the 45-degree line being somewhere between the left limit of the function and the right limit of the function at v^E.

On the one hand, our proof of Theorem 4 shows that the set of equilibria can be fully determined by assuming different tiebreaking rules (i.e., the equilibrium is unique as a function of the probability the indifferent manager discloses, and every equilibrium only involves one indifferent manager and no "upward jump"). On the other hand, a simple argument suggests the left-continuous equilibrium is more compelling. Suppose the manager could, at time 2, change the value from v to $v-\varepsilon$, for some arbitrarily small ε. Insisting that such arbitrarily small devaluations are not uniformly strictly profitable essentially amounts to a requirement that the price function is left continuous. This singles out the equilibrium satisfying the property that an indifferent manager remains silent, as imposed by our model.

4.3 Dynamic Disclosure

To highlight the main forces, Section 4.1 and Second 4.2 assumed that corporate disclosures are either late or early for exogenous reasons. From a modeling perspective, to arrive at these cases we simply assumed that one type of cost is zero and the other is prohibitively high (above the highest firm value of 1). We now return to our main dynamic model which encompasses the most common situations where managers can choose the timing of their disclosure; accordingly, we assume that $c^j \in (0,1)$, $j \in \{E, L\}$.

Theorem 4 shows that the key ideas discussed in relation to purely late and purely early disclosure cases extend naturally to dynamic disclosure. We begin our discussion by pointing out some of the issues that emerge, and clarify how we are able to circumvent each one. We note that avoiding each case uses some features of our model, which we nevertheless viewed as economically relevant.

First, when dynamic disclosure is a possibility, the manager must keep track of the second-period decision, which may be uncertain. When disclosure can only be early as in Section 4.1, all that matters is the *expected* price upon nondisclosure. Furthermore, the fact that this will generally be discontinuous does not prevent existence of a threshold. Intuitively, since in the relevant case the price can only drop *downward* at the discontinuity point, and otherwise always increases at a rate less than 1, there can still only be one intersection point. In the dynamic disclosure case, however, the price itself will influence the incentives of the manager, since she has the *option* to remain silent and delay her decision. We show that the manager's payoff will satisfy an increasing difference property both in the case where the signal is informative as well as in the case where it is not. This uses the fact that the *expected* price given an uninformative signal is independent of the manager's value. Despite the discontinuity, the option value implied at this history does not increase at a rate faster than the value itself (even though this could occur if, say, relevance were exogenous and non-constant in the manager's value).

Second, the equilibrium is more involved when disclosure is dynamic, since the expected payoff from disclosure depends on the market beliefs. One complication this introduces is that we are not guaranteed any differentiability properties of the price function given an arbitrary conjecture regarding the manager's first period strategy, even though this does emerge in equilibrium (except for knife-edge cases). Thus, we use primitive arguments to show that the intuitions outlined in Section 4.1 and Section 4.2 hold generally, and that they then imply the usually increasing differences property.

In what follows, we briefly discuss several straightforward corollaries that consider the relative magnitude of early and late proprietary costs in $(0, 1)$. We begin by briefly noting how proprietary costs affect disclosure incentives.

Corollary 1. *Suppose the thresholds v^E and $v^L(s)$ are uniquely defined (as described in Theorem 5); then they are increasing in $c^E \in (0, 1)$ and $c^L \in (0, 1)$, respectively. There exist cutoffs $\overline{c}^j \in (0, 1)$, $j = \{E, L\}$ such that, if the costs exceed these cutoffs, disclosing early or late, respectively, is never beneficial.*

The intuition is similar to that in Verrecchia (1983): the higher the cost of information disclosure, the larger the set of values for which disclosure is not beneficial and the less negatively the market reacts to the manager's silence. In particular, while $v^E < \mu$ when $c^E = 0$ and $v^L(s) < 1$ when $c^L = 0$, these thresholds can be higher for strictly positive costs and even assume a corner value of 1 (implying nondisclosure for any observed v) when the costs are sufficiently high. For the reminder of the paper we assume none of the costs is prohibitively large, $\overline{c}^j > c^j$ for $j = \{E, L\}$.

Even when the cost is not prohibitively high, the early disclosure threshold can reach

1, depending on the relative differential between the costs. Consider a scenario in which delaying disclosure is weakly cheaper than advancing it, $c^E \geq c^L$ (e.g., because revealing proprietary cost to competitors becomes less damaging as time progresses). A manager who observes $v < v^{s=1} \equiv \min\{1, c^E + P(s = 1, d = \varnothing)\}$ may benefit (but not lose) from delaying her decision: she obtains a market price of v net of c^E by disclosing immediately and a weakly higher price (as the external signal may be favorable and prompt the manager to remain silent) net of $c^L \leq c^E$ if she delays. A manager who observes $v > v^{s=1}$ knows that, if she does not disclose at date 2, she will certainly disclose at date 4, regardless of the external signal. Thus the manager may benefit from delaying—in this case, she obtains the market price of v net of weakly lower cost.

Corollary 2. *It holds that $v^E = 1$ if $c^E \geq c^L$ and $\overline{c}^j > c^j$ where $\overline{c}^j$ for $j = \{E, L\}$ is defined in Corollary 1. That is, all managers at least weakly prefer to delay and disclose at date 4 if $v \geq v^L(s)$, i.e., disclosure of favorable news is clustered at date 4, while there are no disclosures at date 2.*

Now consider the case where disclosing early is cheaper, $c^L > c^E$ (e.g., due to reputation loss for withholding the information early on). At date 2, when deciding whether to delay, an informed manager compares her payoff from sharing her information immediately, $P(s, v) - c^E = v - c^E$, with her expected payoff from delaying the decision to date 4, $\mathbb{E}[P(s, d^E = \varnothing, d^L)|v] - \Pr(d^L = v)c^L$, where d^L represents late disclosure and d^E early disclosure.

Corollary 3. *A threshold equilibrium (as described in Theorem 4 involves $v^E < 1$ if $c^E < c^L$ and $\overline{c}^j > c^j$ where $\overline{c}^j$ for $j = \{E, L\}$ is defined in Corollary 1. Disclosures of favorable information are clustered before the arrival of external news, while disclosures of less favorable information after it.*

Managers observing sufficiently high firm values prefer to disclose early and avoid the higher cost of delaying. The rest prefer to delay their decision until after the arrival of the external signal. Our result aligns with recent empirical evidence about clustering of positive managerial guidance before the release of financial statements by peers and clustering of negative guidance after these releases (Sletten 2012).

The comparative statics of $v^L(s)$ and v^E with respect to q in Sections 4.1 and 4.2, respectively, can shed light on how prior beliefs that external news is relevant impact the timing of corporate disclosures. In particular, external news that is ex ante more likely relevant (higher q) increases the managers' incentives to delay disclosure. The withheld values will be revealed later, unless the external news turns out to be sufficiently favorable.

5 Price Fluctuations with Frequent Adjustments

Our model posits that the market price is formed only once, after all public information is revealed. In capital markets, however, prices are adjusted more frequently. This extension considers price adjustments after the arrival of new information. We assume that disclosure is either only early or only late. Let $P_t^j(\Omega)$ be the price at date t under scenario $j = \{\mathrm{E}, \mathrm{L}\}$ and $\delta \in (0, 1)$ be the discount factor faced by the manager.

Late disclosure with frequently adjusted market price. First, consider the late disclosure case ($c^E > 1$ and $c^L = 0$). The starting price is simply the prior expectation, $P_1^L = \mathbb{E}[v] = \mu$. There is no change until date 3, when the external signal is released and the price is adjusted to $P_3^L(s) = \mathbb{E}[v|s] = qs + (1-q)\mu$. As one would expect, favorable external signals increase the price, and unfavorable ones decrease it; i.e., $P_1^L \lesseqgtr P_3^L(s) \quad \Leftrightarrow \quad s \gtreqless \mu$. At date 4, if the manager discloses, the price is $P_4^L(s, v) = v$. Otherwise, it is $P_4^L(s, \varnothing) = P(s, \varnothing)$. There is no additional information post-disclosure and all future payments remain at their date-4 levels. Thus frequent price adjustment only results in scaling of the same payments that the manager had in Section 4.1 and leaves the manager's decision unaffected.[13]

Corollary 4. *If at date 4 the manager does not disclose, it holds that $P_4^L(s, \varnothing) < P_3^L(s)$ for*

[13]When deciding whether to disclose, the manager compares the present value of her current and future payments from disclosure, $\Pi^L(v) = P_4^L(s, v) + \delta P_5^L(s, v) = (1 + \delta)P(s, v)$, with those from nondisclosure, $\Pi^L(s, \varnothing) = P_4^L(s, \varnothing) + \delta P_5^L(s, \varnothing) = (1 + \delta)P(s, \varnothing)$.

any s. If at date 4 the manager discloses, it holds that $P_4^L(s,v) \lesseqqgtr P_3^L(s)$ if $s \gtreqless \frac{v-(1-q)\mu}{q}$.

When the manager does not respond to the external signal, the price decreases as investors conclude that the manager may have observed an even lower value. However, the price may also decrease when the manager responds (consistent with empirical findings in Sletten 2012). This happens when the manager anticipates a further decrease in the price if she remains silent. To avoid this, the manager discloses values that, while lower than the date-3 price, exceed the price that would have prevailed if she had remained silent.

Early disclosure with frequently adjusted market price. In the case of early disclosure ($c^E = 0$ and $c^L > 1$) it is straightforward that $P_1^E = \mathbb{E}[v] = \mu$ and $P_2^E(v) = v$. The non-disclosure price at date 2, $P_2^E(\varnothing)$, resembles the one defined in (1) but for some conjectured by the investors threshold $\widehat{v}$ instead of v^B. (We derive the equilibrium threshold below.) At date 3, the newly arrived external signal changes the market price only if the manager remained silent at date 2, $P_3^E(s,v) = P_2^E(v) = v$ but $P_3^E(s,\varnothing) \neq P_2^E(\varnothing)$. Market prices at date 4 and 5 remain at their date-3 levels.

Do frequent price adjustments affect the manager's disclosure at date 2? An informed manager discloses if the present value of her payoffs from doing so exceeds that from not.[14]

Corollary 5. *When the market price is frequently adjusted, there exists a threshold $\widetilde{v}^E \in (v^B, v^E)$ such that the manager discloses (early) if and only if $v > \widetilde{v}^E$. The (early) disclosure threshold $\widetilde{v}^E$ is increasing in the discount factor δ.*

Frequent adjustment of stock prices encourages corporate disclosure, $\widetilde{v}^E < v^E$, because the manager faces a lower short-term price at date-2, relative to the expected market price after the signal is realized—this makes pretending to be uninformed less beneficial (in present value terms) and encourages disclosure. Nevertheless, external information continues to suppress corporate disclosure, $\widetilde{v}^E > v^B$. Since the disclosure threshold is increasing in δ,

[14]These present values are $\Pi^E(v) = \mathbb{E}[P_2^E(v)|v] + \delta\mathbb{E}[P_3^E(s,v)|v] + \delta^2\mathbb{E}[P_4^E(s,v)|v] + \delta^3\mathbb{E}[P_5^E(s,v)|v] = (1+\delta+\delta^2+\delta^3)v$ and $\Pi^E(\varnothing) = \mathbb{E}[P_2^E(\varnothing)|v] + \delta\mathbb{E}[P_3^E(s,\varnothing)|v] + \delta^2\mathbb{E}[P_4^E(s,\varnothing)|v] + \delta^3\mathbb{E}[P_5^E(s,\varnothing)v] = P_2^E(\varnothing) + (\delta+\delta^2+\delta^3)\mathbb{E}[P(s,\varnothing)|v]$.

more impatient managers are more likely to disclose their firm value.[15]

It remains to consider the price trend over time. The date-2 disclosure price $P_2^E(v) = v$ exceeds the initial price $P_1^E = \mathbb{E}[v] = \mu$ if $v > \mu$. After disclosure of $v \in [\widetilde{v}^E, \mu)$, the price can decrease (again, consistent with Sletten 2012). This is because the manager expects that the present value of the future price post-silence to be lower. If the manager withholds her information, the date-2 market price always decreases from its initial level, $P_2^E(\varnothing) < P_1^E$.[16] The realization of the external signal affects the date-3 market price in a nontrivial way.

Corollary 6. *The price following disclosure at date 2, $P_2^E(v)$, is lower than the initial price, P_1^E, if the observed and disclosed by the manager value is $v \in [\widetilde{v}^E, \mu)$. There exists $s^{\dagger\dagger} \in (0, \widetilde{v}^E)$ such that: $P_2^E(\varnothing) \geq P_3^E(s, \varnothing)$ if $s \in [0, s^{\dagger\dagger}]$ or $s \in [\widetilde{v}^E, \mu]$. However, $P_2^E(\varnothing) \leq P_3^E(s, \varnothing)$ if $s \in [s^{\dagger\dagger}, \widetilde{v}^E]$ or $s \in [\mu, 1]$.*

Very favorable external news increases the market price, relative to its date-2 level, and extremely unfavorable news decreases the price. However, our result implies that more neutral news (s in the intermediate region) has an ambiguous effect. While the date-2 nondisclosure price $P_2^E(\varnothing) = P(\varnothing)$ does not depend on the signal, the date-3 price, $P_3^E(s, \varnothing) = P(s, \varnothing)$, is non-monotonic in it (Proposition 2). Because news that exceeds the disclosure threshold is perceived to be less likely to be relevant (Proposition 1), the market price around $s = \widetilde{v}^E$ drops in a sense that $\lim_{s \to \widetilde{v}^{\mathrm{E}-}} P_3^E(s, \varnothing) > \lim_{s \to \widetilde{v}^{\mathrm{E}+}} P_3^E(s, \varnothing)$. A signal $s \to \widetilde{v}^{\mathrm{E}+}$ leads to price decrease from its date-2 level. Conversely, a signal $s \to \widetilde{v}^{\mathrm{E}-}$ leads to price increase.

6 Concluding Remarks

We examine how uncertainty surrounding the relevance of external news affects investor beliefs, managers' incentives to reveal private information, and market prices. We find

[15]Extremely impatient managers ($\delta = 0$) disclose only values above v^B: they only care about the current period and not about the external information that will reveal in the next period. Extremely patient managers ($\delta = 1$), however, disclose only values above v^E.

[16]To see why, note that $P_2^E(\varnothing) = P(\varnothing | \widetilde{v}^E) < P(\varnothing | v^B) = v^B < \mu = P_1^E$.

that investors perceive favorable external news as less likely relevant, which reinforces their belief that managers possess unfavorable information. As a result, investor beliefs depend on the external news and are discontinuous; furthermore, there is a possibility of price non-monotonicity where relatively more favorable external news could paradoxically lead to a lower market price. We also find that the timing, likelihood for relevance and content of external news can either encourage or discourage corporate disclosures.

Our analysis has practical implications for investors and firms deciding whether to disclose private information in an environment with abundant external news like media broadcasts, analyst forecasts, peer reports, and macroeconomic news. The results may provide insights into the findings of recent empirical work in this area, such as Sletten (2012) and Capkun, Lou, Otto, and Wan (2022). Furthermore, our research suggests that market prices may have non-monotonic behavior and non-constant sensitivity to external news—a finding that may be related to empirical studies on asymmetric asset price reactions (e.g., Aggarwal and Schirm 1998; Goldberg and Grisse 2013; Blot, Hubert and Labondance 2020; Xu and You 2023). Future empirical research may account for the uncertainty in the relevance of external news when investigating firm disclosure behavior and price reactions.

Appendix: Proofs and Supplemental Analysis

Proof of Lemma 1: Follows directly from the proof in Dye (1985) and Jung and Kwon (1998) and is omitted.

Proof of Lemma 2: We consider the joint belief $\Pr(\rho, \kappa | s, \varnothing)$ separately for the cases $s < \widehat{v}$ and $s > \widehat{v}$.

<u>Case 1:</u> Let the observed signal be $s = s' \in [0, \widehat{v})$. Because s is a continuous variable, we use the limit of measurable intervals and express $\Pr(\rho, \kappa | s = s', \varnothing) = \lim_{\Delta \to 0} \frac{\Pr(s \in [s', s'+\Delta], \varnothing, \rho, \kappa)}{\Pr(s \in [s', s'+\Delta], \varnothing)}$ for $\rho \in \{N, R\}$, $\kappa \in \{I, U\}$ and $\Delta > 0$ sufficiently small so that $s' + \Delta \in [0, \widehat{v})$. Let us define $A' \equiv [s', s' + \Delta]$ and focus on the numerator. We note that:

$$\begin{aligned}
\Pr(s \in A', \varnothing, R, I) &= p \cdot q \cdot 1 \cdot \Pr(s \in A'); \\
\Pr(s \in A', \varnothing, R, U) &= (1-p) \cdot 1 \cdot q \cdot \Pr(s \in A'); \\
\Pr(s \in A', \varnothing, N, I) &= p \cdot G(\widehat{v}) \cdot (1-q) \cdot \Pr(s \in A'); \\
\Pr(s \in A', \varnothing, N, U) &= (1-p) \cdot (1-q) \cdot \Pr(s \in A').
\end{aligned}$$

For the denominator in $\Pr(\rho, \kappa | s = s', \varnothing)$, we have $\Pr(s \in A', \varnothing) = \Pr(s \in A', \varnothing, R, I) + \Pr(s \in A', \varnothing, R, U) + \Pr(s \in A', \varnothing, N, I) + \Pr(s \in A', \varnothing, N, U) = \Big(q + (1-q)\big(1 - p + pG(\widehat{v})\big)\Big) \cdot \Pr(s \in A')$. Now we can express:

$$\begin{aligned}
\Pr(U, R | s = s', \varnothing) &= \lim_{\Delta \to 0} \frac{\Pr(s \in A', \varnothing, R, U)}{\Pr(s \in A', \varnothing)} \\
&= \frac{(1-p)q}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A')}}{\cancel{\Pr(s \in A')}}; \\
\Pr(U, N | s = s', \varnothing) &= \lim_{\Delta \to 0} \frac{\Pr(s \in A', \varnothing, N, U)}{\Pr(s \in A', \varnothing)} \\
&= \frac{(1-p)(1-q)}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A')}}{\cancel{\Pr(s \in A')}}; \\
\Pr(I, R | s = s', \varnothing) &= \lim_{\Delta \to 0} \frac{\Pr(s \in A', \varnothing, R, I)}{\Pr(s \in A', \varnothing)} \\
&= \frac{pq}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A')}}{\cancel{\Pr(s \in A')}}; \\
\Pr(I, N | s = s', \varnothing) &= \lim_{\Delta \to 0} \frac{\Pr(s \in A', \varnothing, N, I)}{\Pr(s \in A', \varnothing)} \\
&= \frac{pG(\widehat{v})(1-q)}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A')}}{\cancel{\Pr(s \in A')}}.
\end{aligned}$$

Case 2: Let the observed signal be $s = s'' \in (\widehat{v}, 1]$. Defining $A'' \equiv [s'', s'' + \Delta]$ for $\Delta > 0$ sufficiently small, so that $s'' + \Delta \in (\widehat{v}, 1]$, and following similar steps,

$$\begin{aligned}
\Pr(s \in A'', \varnothing, R, I) &= 0 \cdot p \cdot G(\widehat{v}) \cdot \Pr(s \in A''); \\
\Pr(s \in A'', \varnothing, R, U) &= (1-p) \cdot q \cdot \Pr(s \in A''); \\
\Pr(s \in A'', \varnothing, N, I) &= p \cdot G(\widehat{v}) \cdot (1-q) \cdot \Pr(s \in A''); \\
\Pr(s \in A'', \varnothing, N, U) &= (1-p) \cdot (1-q) \cdot \Pr(s \in A'').
\end{aligned}$$

Summing up, we have $\Pr(s \in A'', \varnothing) = \Big(q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)\Big) \cdot \Pr(s \in A'')$. We can express

$$\begin{aligned}
\Pr(U, R | s = s'', \varnothing) &= \frac{(1-p)q}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A'')}}{\cancel{\Pr(s \in A'')}}; \\
\Pr(U, N | s = s'', \varnothing) &= \frac{(1-p)(1-q)}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A'')}}{\cancel{\Pr(s \in A'')}}; \\
\Pr(I, R | s = s'', \varnothing) &= 0; \\
\Pr(I, N | s = s'', \varnothing) &= \frac{pG(\widehat{v})(1-q)}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A'')}}{\cancel{\Pr(s \in A'')}}.
\end{aligned}$$

Summarizing yields our result.

We briefly mention that the same argument applies in cases where the first-period disclosure strategy is not characterized by a threshold, simply by modifying the distribution G appropriately.

Proof of Theorem 1: Follows from Lemma 1.

Proof of Lemma 3: Follows directly from equation (4) and Lemma 2.

Proof of Theorem 2: Follows from Lemma 3.

Proof of Theorem 3: Follows from Lemma 3.

Proof of Theorem 4: Included in Section A of this Appendix.

Proof of Theorem 5: Included in Section A of this Appendix.

Proof of Theorem 6: Follows from Propositions 2 and 3.

Proof of Proposition 1: We consider the comparative static, given the threshold equilibrium. Note that, since $c^E > 1$ and $c^L = 0$, we have the following equation (which also uses the fact that s provides no information about v if irrelevant and is equal to v if relevant):

$$v^L(s) = s\Pr(R|s,\varnothing) + \mathbb{E}[v|N,\varnothing]\Pr(N|s,\varnothing)$$

Suppose that $s > v^B$. Then by definition of v^B and Lemma A.12, values that equal to relevant signals are disclosed, i.e., $v = s > v^L(s)$. Thus, it must be that $s > \mathbb{E}[v|N,\varnothing]$, since otherwise a manager with value equal to s would have a profitable deviation to remain silent. Since increasing q increases the weight on s and decreases the weight on $\mathbb{E}[v|N,\varnothing]$, we have that the right hand side increases, meaning that $v^L(s)$ increases as well.

On the other hand, if $s < v^B$, the opposite conclusions hold, implying that as the weight on s increases, the threshold decreases.

Lemma A.12 and A.13 imply that $v^L(s)$ is increasing at a rate less than 1, since $\widehat{v}_0^L(s)$ and $\widehat{v}_1^L(s)$ both are and $v^L(s)$ is equal to the latter for $s > v^B$ and the former for $s < v^B$. So when $s < v^B$ it holds that $v^L(s) > s$; however, since the $v^L(s)$ is decreasing in s, we also have $v^L(s) < v^B$. The other cases are analogous.

Proof of Proposition 2: Note that Lemma A.12 shows that $\widehat{v}_1^L(s) - \widehat{v}_0^L(s)$ is decreasing in s, which immediately implies the second part of the claim. Continuity follows from Lemma A.11, together with the observation that since $v^L(s) = \min\{\widehat{v}_1^L(s), \widehat{v}_0^L(s)\}$, and the minimum of two continuous functions is itself continuous.

Proof of Proposition 3: The proof that $v^E \geq v^B$ follows from Lemma A.6 and Proposition 2. To see this note that if $P(s,\varnothing)$ has a discontinuous jump downwards at some $s = \widehat{v} > v^B$, then $\mathbb{E}[P(s,\varnothing)|v]$ has a discontinuous jump downwards at $v = \widehat{v} > v^B$. We now argue this threshold must be strictly above v^B.

Slightly abusing notation, let $P^q(s, v = \widehat{v})$ denote the price when the market conjectures threshold $\widehat{v}$, given relevance q. We first claim that, at $\widehat{v} = v^B$, this function is strictly convex in q, at any $s \neq v^B$. We also claim that $\frac{d}{dq}\int_0^1 P^q(s, v = \widehat{v})g(s)ds$ is equal to 0 at $q = 0$.

Now, note that $P^q(v^B, v^B = \widehat{v}) = v^B$. Also note that $\int_0^1 P^0(s, v^B = \widehat{v})g(s)ds = v^B$, and that putting the two claims above together, we also have that $\int_0^1 P^q(s, v^B = \widehat{v})g(s)ds$ is strictly increasing in q, for all $q > 0$; indeed, for almost every s, $P^q(s, v^B = \widehat{v})$ is strictly convex, meaning that the expectation over s is strictly convex as well. Since the derivative is 0 at $q = 0$, we have this function is strictly increasing.

So consider the manager's incentives. If the market conjectured a threshold $\widehat{v} = v^B$, the manager at the threshold would obtain v^B from disclosing, but:

$$qP^q(v^B, v^B = \widehat{v}) + (1-q)\int_0^1 P^q(s, v^B = \widehat{v})g(s)ds$$

from not disclosing. However, we have just seen that this is a convex combination of v^B and a term which is strictly larger than v^B. Thus, the manager strictly gains from not disclosing, showing that we must have $\widehat{v} > v^B$.

To compare v^E and μ, we use that $\lim_{\widehat{v}\to\mu} \mathbb{E}_s[P(s, v = \widehat{v})] = \mu$ while $\lim_{\widehat{v}\to\mu} \mathbb{E}_s[P(s, \varnothing)|v] < \mu$; to see this point, we analyze the price as a function of the signal:

$$P(s, \varnothing) = \frac{qs(1 - p\mathbb{1}_{s>\widehat{v}}) + (1 - q)((1 - p)\mu + p\int_0^{\widehat{v}} vg(v)dv)}{q(1 - p\mathbb{1}_{s>\widehat{v}}) + (1 - q)(1 - p + pG(\widehat{v}))}.$$

Note that, if the signal is relevant and the manager's value is $\widehat{v}$, then $(1-p)\mu+p\int_0^{\widehat{v}} vg(v)dv) < \widehat{v}$ whenever $\widehat{v} > v^B$. Thus, at $\widehat{v} = \mu$, the price is less than μ if the signal is relevant. On the other hand, if $s \sim G$, note that we have, if $s > \widehat{v}$, that then $(1 - p)\mu + p\int_0^{\widehat{v}} vg(v)dv < \widehat{v} < s$, meaning that the price would be even larger if the market conjectured such signals could have come from informed managers; that is,

$$P(s, \varnothing) \leq \frac{qs + (1 - q)((1 - p)\mu + p\int_0^{\widehat{v}} vg(v)dv)}{q(1 - p) + (1 - q)(1 - p + pG(\widehat{v}))}$$

Now, when we take the expectation of the right hand side of this expression, we have:

$$\frac{q\mu + (1 - q)((1 - p)\mu + p\int_0^{\widehat{v}} vg(v)dv)}{q(1 - p) + (1 - q)(1 - p + pG(\widehat{v}))} < \mu, \forall\widehat{v}.$$

Thus, we see that if $\widehat{v} = \mu$, the price is (bounded above by) a convex combination of terms that are equal to μ and those that are a strictly less than μ, as claimed.

We now return to the two claims above regarding $P^q(s, v = \widehat{v})$. The claim on strict convexity involves some tedious algebra, which reveals that $\frac{\partial^2}{\partial q^2}P^q(s, \widehat{v}) > 0$ if and only if:

$$(\mathbb{1}_{s>\widehat{v}} - 1 + G(\widehat{v}))((1 - p)(s - \mu) + p\int_0^{\widehat{v}} (s - v)g(v)dv) > 0.$$

We note that:

$$0 = (1 - p)(s - \mu) + p\int_0^{\widehat{v}} (s - v)g(v)dv) \Leftrightarrow s = \frac{\mu(1 - p) + p\int_0^{\widehat{v}} vg(v)dv}{1 - p + pG(\widehat{v})}.$$

If $\widehat{v} = v^B$, then this holds if and only if $s = v^B$. So, this term is negative if $s < v^B$ and positive if $s > v^B$. Similarly, $\mathbb{1}_{s>v^B} - 1 + G(v^B)$ is positive for $s > v^B$ and negative for $s < v^B$. Putting this together, we have strict convexity holds.

As for the second claim, we note that we can bring the derivative inside the expectation and have:

$$\frac{\partial}{\partial q}\bigg|_{q=0}\int_0^1 P^q(s,v=\widehat{v})g(s)ds \propto \int_0^1 \bigg((1-p+pG(\widehat{v}))(s(1-p\mathbb{1}_{s>\widehat{v}})-(1-p)\mu-p\int_0^{\widehat{v}} vg(v)dv)$$

$$-((1-p)\mu+p\int_0^{\widehat{v}} vg(v)dv)(1-p\mathbb{1}_{s>\widehat{v}}-((1-p)+pG(\widehat{v})))\bigg)g(s)ds.$$

This can be obtained by noting that the denominator of $\frac{\partial}{\partial q}\Big|_{q=0} P^q(s,v=\widehat{v})$ is $(1-p+pG(\widehat{v}))^2$ (and hence independent of s, meaning it can be dropped without changing the sign of the derivative); the value of the denominator of $P^q(s,v=\widehat{v})$ at $q=0$ is $1-p+pG(\widehat{v})$; the derivative of the denominator with respect to q evaluated at $q=0$ is $1-p\mathbb{1}_{s>\widehat{v}}-(1-p+pG(\widehat{v}))$; the value of the numerator at $q=0$ is $(1-p)\mu+p\int_0^{\widehat{v}} vg(v)dv$; the derivative at $q=0$ is $s(1-p\mathbb{1}_{s>\widehat{v}})-(1-p)\mu-p\int_0^{\widehat{v}} vg(v)dv$. That this expression is equal to 0 follows from the observation that:

$$\int_0^1 \left(s(1-p\mathbb{1}_{s>\widehat{v}})-(1-p)\mu-p\int_0^{\widehat{v}} vg(v)dv\right)g(s)ds=0$$

$$=\int_0^1 \left((1-p\mathbb{1}_{s>\widehat{v}}-((1-p)+pG(\widehat{v}))\right)g(s)ds,$$

completing the proof of the second claim; to see these equalities, note that $1-p\mathbb{1}_{s>\widehat{v}}=1-p+p\mathbb{1}_{s\le\widehat{v}}$.

Proof of Corollary 1: The proof follows similar steps to Verrecchia (1983) and is omitted.

Proof of Corollary 2: The proof follows from the discussion in the text and is omitted.

Proof of Corollary 3: The proof follows from the discussion in the text and is omitted.

Proof of Corollary 4: The comparison between $P_4^L(s,v)$ and $P_3^L(s)$ is straightforward. Here, we only compare $P_4^L(s,\varnothing)$ with $P_3^L(s)$. Recall that, $P_3^L(s)=qs+(1-q)\mu$ and $P_4^L(s,\varnothing)=\widetilde{v}^L(s)$. Applying the Envelope Theorem, $\frac{d}{ds}\widetilde{v}^L(s)=\frac{1-p}{1-p+pG(\widetilde{v}^L(s))}\cdot q+\frac{pG(\widetilde{v}^L(s))}{1-p+pG(v^L(s))}\cdot\frac{q}{q+(1-q)G(\widetilde{v}^L(s))}>q=\frac{d}{ds}P_3^L(s)$.

Let s^o be the signal realization that satisfies $s^o=v^L(s=s^o)$. We note that $P_3^L(s=s^o)=q\cdot s^o+(1-q)\mu>s^o=v^L(s=s^o)=P_4^L(s=s^o,\varnothing)$, because $s^o<\mu$. Hence, we have $P_3^L(s)>P_4^L(s,\varnothing)$ for any $s\ge s^o$. It remains to show that this inequality holds for $s<s^o$.

We note that

$$\begin{aligned}
\min P_4^L(s,\varnothing) &= P_3^L(s=0,\varnothing) \\
&= \frac{(1-p)(1-q)\mu + pG(\widetilde{v}^L(s=0))\left(1-\frac{q}{q+(1-q)G(\widetilde{v}^L(s=0))}\right)\mathbb{E}[v|v\le \widetilde{v}^L(s=0)]}{1-p+pG(\widetilde{v}^L(s=0))} \\
&< \frac{(1-p)(1-q)\mu + pG(\widetilde{v}^L(s=0))\,(1-q)\,\mathbb{E}[v|v\le \widetilde{v}^L(s=0)]}{1-p+pG(\widetilde{v}^L(s=0))} \\
&< (1-q)\mu = P_3^L(s=0) = \min P_3^L(s).
\end{aligned}$$

Therefore, $P_4^L(s,\varnothing) < P_3^L(s)$ for any s.

Proof of Corollary 5: The equilibrium condition is $\Delta\Pi(\widehat{v}=\widetilde{v}^E) = \Pi(v) - \Pi(\varnothing) = 0$. Using the notation in the proof of Proposition 3, we can simplify,

$$\Delta\Pi(\widehat{v}) = T^B(\widehat{v}) + (\delta+\delta^2+\delta^3)T^E(\widehat{v}).$$

Both $T^B(\widehat{v})$ and $T^E(\widehat{v})$ are decreasing in $\widehat{v}$. Thus, $\Delta\Pi(\widehat{v})$ is also decreasing. Furthermore,

$$\begin{aligned}
\lim_{\widehat{v}\to 1}\Delta\Pi(\widehat{v}) &= \underbrace{\lim_{\widehat{v}\to 1}T^B(\widehat{v})}_{<0} + (\delta+\delta^2+\delta^3)\underbrace{\lim_{\widehat{v}\to 1}T^E(\widehat{v})}_{<0} < 0; \\
\lim_{\widehat{v}\to 0}\Delta\Pi(\widehat{v}) &= \underbrace{\lim_{\widehat{v}\to 0}T^B(\widehat{v})}_{>0} + (\delta+\delta^2+\delta^3)\underbrace{\lim_{\widehat{v}\to 0}T^E(\widehat{v})}_{>0} > 0.
\end{aligned}$$

Therefore, there exists a unique threshold $\widetilde{v}^E \in (0,1)$ such that the manager discloses if and only if $v > \widetilde{v}^E$. To see that $\widetilde{v}^E \in (v^B, v^E)$ note that

$$\begin{aligned}
\lim_{\widehat{v}\to v^B}\Delta\Pi(\widehat{v}) &= \underbrace{\lim_{\widehat{v}\to v^B}T^B(\widehat{v})}_{=0} + (\delta+\delta^2+\delta^3)\underbrace{\lim_{\widehat{v}\to v^B}T^E(\widehat{v})}_{>0} > 0; \\
\lim_{\widehat{v}\to v^E}\Delta\Pi(\widehat{v}) &= \underbrace{\lim_{\widehat{v}\to v^E}T^B(\widehat{v})}_{<0} + (\delta+\delta^2+\delta^3)\underbrace{\lim_{\widehat{v}\to v^E}T^E(\widehat{v})}_{=0} < 0.
\end{aligned}$$

Lastly, using the Implicit Function Theorem,

$$\frac{\partial}{\partial\delta}v^E \propto \frac{\partial}{\partial\delta}\Delta\Pi(\widetilde{v}^E) = (1+2\delta+3\delta^2)\underbrace{T^E(\widehat{v}=\widetilde{v}^E)}_{>0} > 0.$$

Proof of Corollary 6: The first part follows from the discussion in the text and is omitted.

For the second part, we know that $P_2^E(\varnothing|\widetilde{v}^E) - P_3^E(s,\varnothing|\widetilde{v}^E) = P(\varnothing|\widetilde{v}^E) - P(s,\varnothing|\widetilde{v}^E)$, where

$$\begin{aligned}
P(\varnothing|\widetilde{v}^E) &= \frac{1-p}{1-p+p\cdot G(\widetilde{v}^E)}\cdot\mu + \frac{p\cdot G(\widetilde{v}^E)}{1-p+p\cdot G(\widetilde{v}^E)}\cdot\mathbb{E}[v|v\leq\widetilde{v}^E] \\
P(s,\varnothing|\widetilde{v}^E, s\leq\widetilde{v}^E) &= \frac{1-p}{1-p+p\cdot G(\widetilde{v}^E)}[(1-q)\cdot\mu + q\cdot s] \\
&\quad + \frac{p\cdot G(\widetilde{v}^E)}{1-p+p\cdot G(\widetilde{v}^E)}[(1-q\cdot\pi(\widetilde{v}^E))\cdot\mathbb{E}[v|v\leq\widetilde{v}^E] + q\cdot\pi(\widetilde{v}^E)\cdot s] \\
P(s,\varnothing|\widetilde{v}^E, s>\widetilde{v}^E) &= \frac{1-p}{1-p+p\cdot G(\widetilde{v}^E)}[(1-q)\cdot\mu + q\cdot s] \\
&\quad + \frac{p\cdot G(\widetilde{v}^E)}{1-p+p\cdot G(\widetilde{v}^E)}\cdot\mathbb{E}[v|v\leq\widetilde{v}^E].
\end{aligned}$$

Therefore:

$$P_2^E(\varnothing|\widetilde{v}^E, s>\widetilde{v}^E) - P_3^E(s,\varnothing|\widetilde{v}^E, s>\widetilde{v}^E) \quad\propto\quad \mu - (1-q)\cdot\mu - q\cdot s \propto \mu - s,$$

which is positive for $s<\mu$ and negative otherwise. Furthermore,

$$\begin{aligned}
w(s) &\equiv P_2^E(\varnothing|\widetilde{v}^E, s\leq\widetilde{v}^E) - P_3^E(s,\varnothing|\widetilde{v}^E, s\leq\widetilde{v}^E) \\
&= \frac{1-p}{1-p+p\cdot G(\widetilde{v}^E)}\cdot(\mu - s) + \frac{p\cdot G(\widetilde{v}^E)}{1-p+p\cdot G(\widetilde{v}^E)}\pi(\widetilde{v}^E)(\mathbb{E}[v|v\leq\widetilde{v}^E] - s)
\end{aligned}$$

This term is decreasing in s. It is immediate that $w(s=0)>0$. Furthermore,

$$w(s=\widetilde{v}^E) = \frac{1-p}{1-p+p\cdot G(\widetilde{v}^E)}\cdot(\mu-\widetilde{v}^E) + \frac{p\cdot G(\widetilde{v}^E)}{1-p+p\cdot G(\widetilde{v}^E)}\pi(\widetilde{v}^E)(\mathbb{E}[v|v\leq\widetilde{v}^E] - \widetilde{v}^E).$$

From the proof of Proposition 5 recall that $\widetilde{v}^E$ satisfies $\Delta\Pi(\widehat{v}=\widetilde{v}^E)=0$. Rearranging,

$$\begin{aligned}
\Delta\Pi(\widehat{v}=\widetilde{v}^E) &= (1+\delta+\delta^2+\delta^3)(v - \mathbb{E}[P(s,\varnothing)|v=\widehat{v},\widehat{v}=\widetilde{v}^E]) \\
&\quad + \mathbb{E}[P(s,\varnothing)|v=\widehat{v},\widehat{v}=\widetilde{v}^E] - P(\varnothing|\widehat{v}=\widetilde{v}^E).
\end{aligned}$$

Because $\widetilde{v}^E\in(v^B,v^E)$, it holds that $(1+\delta+\delta^2+\delta^3)(v-\mathbb{E}[P(s,\varnothing)|v=\widehat{v},\widehat{v}=\widetilde{v}^E])<0$. Therefore, it has to be that $\mathbb{E}[P(s,\varnothing)|v=\widehat{v},\widehat{v}=\widetilde{v}^E] - P(\varnothing|\widehat{v}=\widetilde{v}^E)>0$. Lastly, note that

$$w(s=\widetilde{v}^E) = -(\mathbb{E}[P(s,\varnothing)|v=\widehat{v},\widehat{v}=\widetilde{v}^E] - P(\varnothing|\widehat{v}=\widetilde{v}^E)) < 0.$$

Therefore, there exists $s^{\dagger\dagger}\in(0,\widetilde{v}^E)$ such that: $P_2^E(\varnothing)\geq P_3^E(s,\varnothing)$ if $s\in[0,s^{\dagger\dagger}]$ or $s\in[\widetilde{v}^E,\mu]$. However, $P_2^E(\varnothing)\leq P_3^E(s,\varnothing)$ if $s\in[s^{\dagger\dagger},\widetilde{v}^E]$ or $s\in[\mu,1]$.

Supplemental Analysis

A Detailed Proof of Theorems 4 and 5

We introduce some notation to allow for more general strategies beyond those considered in the main text, in particular those that may allow for mixed strategies. Note that an arbitrary strategy for the manager can be represented as

$$\sigma^E : [0,1] \to \Delta(\{Disclosure, Silence\}), \sigma^L : [0,1] \times [0,1] \to \Delta(\{Disclosure, Silence\}),$$

where σ^E specifies the early disclosure decision while σ^L specifies the late disclosure decision; the former depends only on the firm value, whereas the latter depends on both the firm value as well as the external signal. In particular, since the manager's strategy is irrelevant after disclosure occurs, the second period strategy does not need to condition on the first period action (i.e. we take this to be "silence" by assumption). Slightly abusing notation let the induced equilibrium price function be $P^{\sigma^E,\sigma^L}(s, \varnothing)$.

The proof of existence and uniqueness make use of some important details of equilibrium behavior of our environment and thus involve some substantial detours. Therefore, we describe how each part comes together to yield proofs of the theorem in Section A.5, where we also highlight some additional implications of our analysis.

Before turning to the proof, we present a result of independent interest, for the sake of completeness: that with a log-concave value distribution and an intermediate probability that the manager is informed, there is a unique threshold equilibrium in the baseline model. That this holds without uncertainty about the information endowment of the manager can be seen from Bagnoli and Bergstrom (2005).

Lemma A.1. *In the baseline model without external signals, if g is log-concave, then $\mathbb{E}[v|v \leq \overline{v}]$ is increasing in $\overline{v}$ at a rate less than 1.*

Proof of Lemma A.1. Computing the derivative of $\mathbb{E}[v|v \leq \overline{v}]$ with respect to $\overline{v}$, we see that it is:

$$\frac{pg(\overline{v})(\overline{v} - \mathbb{E}[v|v \leq \overline{v}])}{(1 - p + pG(\overline{v}))}$$

If $\overline{v} < v^B$, then this expression is negative, and hence less than 1 for all p such that $\overline{v} < v^B$. Otherwise, for all $\overline{v}$, the derivative of this expression with respect to p is proportional to:

$$(1-p+pG(\overline{v}))\left(g(\overline{v})(\overline{v}-\mathbb{E}[v|v\leq\overline{v}])-pg(v)\frac{\partial}{\partial p}\mathbb{E}[v|v\leq\overline{v}]\right)+pg(\overline{v})(\overline{v}-\mathbb{E}[v|v\leq\overline{v}])(1-G(\overline{v})).$$

If $\overline{v} > \mathbb{E}[v \mid v \leq \overline{v}]$, then it is immediate that this expression is positive for all p (an observation that uses $\mathbb{E}[v|v \leq \overline{v}]$ is decreasing in p; see Kartik, Lee and Suen (2019) and references cited therein); thus, since the slope is increasing in p, it is less than the slope in the case that $p = 1$; since Bagnoli and Bergstrom (2005) implies the slope is less than 1 in this case, we have the slope is less than 1 for all p, as claimed. □

A.1 Preliminaries

We start with some preliminary observations which play a role in various parts of the proof; first, that *late* disclosure must be characterized a threshold which depends on s; second, that beliefs about relevance have no impact on market price only for a single signal; and third, that the price (as a function of the signal) has slope less than 1 which is flattened as the market assigns lower probability to relevance. The last point makes use of an auxiliary game which we introduce and discuss later in the proof.

Lemma A.2. *If σ^L is an equilibrium late disclosure strategy, then σ^L is characterized by a threshold, $v^{L,\sigma_E}(s)$ such that the manager discloses whenever $s > v^{L,\sigma_E}(s)$.*

Proof of Lemma A.2. Standard; the manager's payoff from non-disclosure is constant in equilibrium, whereas the payoff from disclosure is increasing in the manager's value. □

Lemma A.3. *Given any arbitrary σ^E, there exists a single signal, which we denote s^B, such that the expectation of v conditional on s^B and non-disclosure is constant in the belief about relevance.*

Proof of Lemma A.3. The signal can be solved for in closed form; if the market believes the relevant signals are never disclosed:

$$s^B = \frac{qs^B + (1-q)\left((1-p)\mu + p\int_0^1 v\sigma^E(v)g(v)dv\right)}{q + (1-q)(1-p) + p\int_0^1 \sigma^E(v)g(v)dv}. \tag{6}$$

We can then solve for s^B as:

$$s^B = \frac{(1-p)\mu + p\int_0^1 v\sigma^E(v)g(v)dv}{(1-p) + p\int_0^1 \sigma^E(v)g(v)dv}.$$

We note that, varying the market belief that relevant signals are disclosed is equivalent to changing the value of q in equation (6). But as we have seen, the value for s^B does not depend on q, and furthermore this value is always uniquely defined. □

A.1.1 The Auxiliary Game

We now describe the auxiliary game which will be useful in our analysis. This game is identical to our main model, but if the manager does not disclose early, then the late-disclosure strategy is assumed to be exogenous whenever the signal is relevant. That is, we assume that if $\rho = R$, the manager discloses with probability ϵ if informed. We denote $\widehat{\sigma}^{L\epsilon}$ as a candidate late disclosure strategy in the auxiliary game. When discussing the auxiliary game, we restrict to cases where the threshold $\widehat{v}^L_\epsilon(s)$ is continuous in s; existence for such equilibria is established in Lemma A.11.

Lemma A.4. *Let $P^{\sigma^E,\sigma^L,\epsilon}(s,\varnothing)$ correspond to the price function in the auxiliary game when the manager uses strategies σ^E and σ^L. The slope of the price function is less than 1, for any ϵ.*

Proof of Lemma A.4. We first show that $\widehat{v}^L_\epsilon(s)$ increases at a rate less than 1. Consider the implicit condition

$$0 = \widehat{v}^L_\epsilon(s) - c^L - \frac{q(1-p\epsilon)s + (1-q(1-p\epsilon))\left((1-p)\mu + p\int_0^{\widehat{v}^L_\epsilon(s)} v\sigma^E(v)g(v)dv\right)}{q(1-p\epsilon) + (1-q(1-p\epsilon))\left((1-p)\int_0^1 g(v)dv + p\int_0^{\widehat{v}^L_\epsilon(s)} \sigma^E(v)g(v)dv\right)}. \quad (7)$$

In particular, the argument that the ratio in Equation (7) defines the expected value upon nondisclosure follows the same argument as in Proposition 1. Suppose that s increases to $s+\Delta$ for Δ small. Then the change in $\widehat{v}^L_\epsilon(s)$, say $\widetilde{\Delta}$, is equal to the change in the ratio defined in (7). Since $\widehat{v}^L_\epsilon(s)$ is continuous, we know that this change must be small as well. If s increases by Δ, this ratio becomes:

$$\frac{q(1-p\epsilon)(s+\Delta) + (1-q(1-p\epsilon))\left((1-p)\int_0^1 vg(v)dv + p\int_0^{\widehat{v}^L_\epsilon(s)+\widetilde{\Delta}} v\sigma^E(v)g(v)dv\right)}{q(1-p\epsilon) + (1-q(1-p\epsilon))\left((1-p)\int_0^1 g(v)dv + p\int_0^{\widehat{v}^L_\epsilon(s)+\widetilde{\Delta}} \sigma^E(v)g(v)dv\right)}$$

Noting that we must have $\widetilde{\Delta}$ small, we obtain the following approximation for $\widetilde{\Delta}$, using (7) (which becomes increasingly accurate as $\Delta \to 0$):

$$\widetilde{\Delta} \approx \frac{q(1-p\epsilon)\Delta + (1-q)p\widetilde{\Delta}\widehat{v}^L_\epsilon(s)\sigma^E(\widehat{v}^L_\epsilon(s))g(\widehat{v}^L_\epsilon(s))}{q(1-p\epsilon) + (1-q)\left((1-p)\mu + p\int_0^{\widehat{v}^L_\epsilon(s)+\widetilde{\Delta}} g(v)\sigma^E(v)dv\right)}.$$

(An upper bound of this quantity can be obtained by taking $\widetilde{\Delta} = 0$ in the denominator, and the argument will still work) Note that the denominator is greater than q. Thus, without the $\widetilde{\Delta}$ terms in the numerator on the right hand side, $\widetilde{\Delta}$ would increase by less than $(q/q) \cdot \Delta$; adding these terms in, we see the necessary increase in $\widetilde{\Delta}$ is even smaller. Thus, $\widetilde{\Delta} < \Delta$. The result follows from noting that the implicit conditional also defines the rate of change of the price as a function of the signal, provided this implicit condition is satisfied; if note, then this threshold does not adjust and an identical argument can be used (simply treating $\widetilde{\Delta} = 0$). □

Lemma A.5. *The slope of $P^{\sigma^E,\sigma^L,\epsilon}(s, \varnothing)$ is decreasing in ϵ.*

Proof of Lemma A.5. Inspecting (7), we see that as ϵ increases, less weight is placed on s and more is placed on a term that is independent of s, implying that the price as a function of s is flatter when ϵ is larger. □

Putting together Lemmas A.3, and A.5 we see that varying ϵ essentially "twists" the price through s^B.

A.2 Early Disclosure Must Be a Threshold

In this section, we show that the early disclosure decision *must* be characterized by a threshold. The argument is more involved than the case of late disclosure, since we need to take into account the possibility of option value.

Lemma A.6. *If σ^E prescribes that the manager discloses early with probability 1 when the value is v, then $v \geq s^B$.*

Proof of Lemma A.6. Suppose to the contrary, and that in fact some $v < s^B$ discloses with probability 1 under σ^E. Note that a manager with value 0 gets payoff 0 from disclosing, and positive payoff from never disclosing (due to the positive probability of being uninformed); the same is true for all managers with values sufficiently close to 0. So, if v^{σ^E} is the set of manager values which disclose under σ^E, and $v^* = \inf v^{\sigma^E}$, then we can find a sequence $v^n_\emptyset \to \widetilde{v}$ such that $v^n_\emptyset$ prefers not disclosing early whereas $\widetilde{v}$ does, for some $\widetilde{v}$. Note that, since $\widetilde{v} < s^B$, we have $\widehat{v}^L_1(\widetilde{v}) > \widehat{v}^L_0(\widetilde{v})$. On the other hand, the expected payoff conditional on the signal being irrelevant is constant in the manager's value.

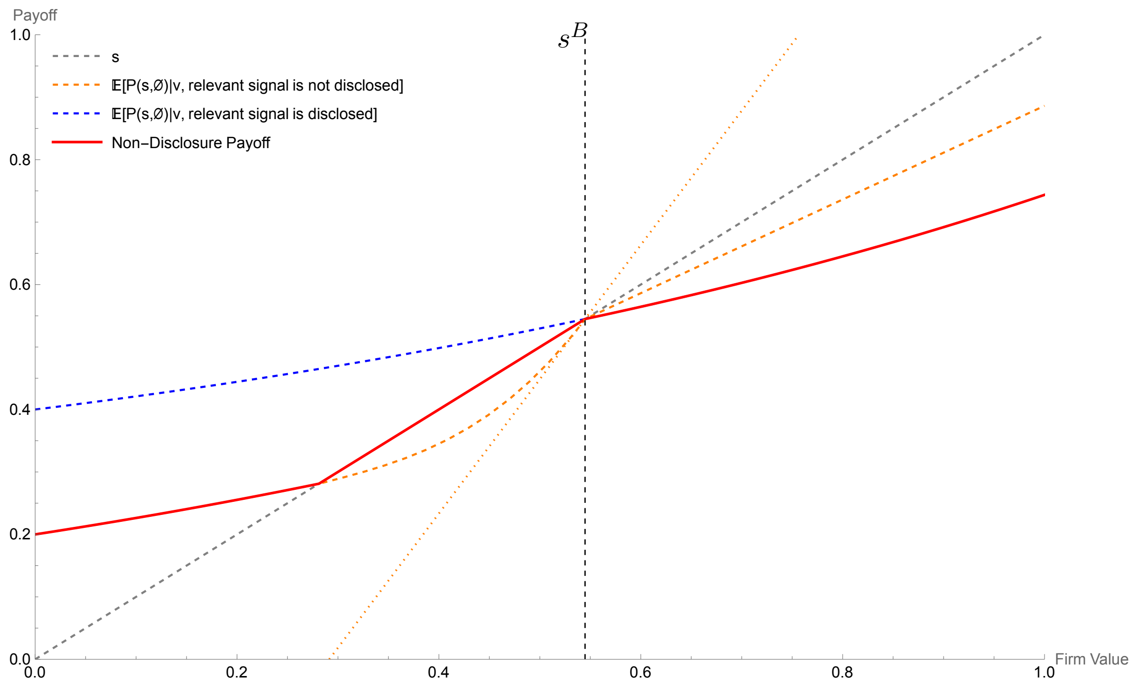

Figure 8: Graphical Proof of Lemma A.7; if there are two points such that the price does not depend on the market's conjecture regarding whether a signal that is relevant would have been disclosed by the manager—which must be the case if the manager is indifferent between decisions at some value below $v^* < s^B$—then there must be some point in (v^*, s^B) at which the slope of the expected price given v is greater than 1.

So, compare the manager's payoffs under $\widetilde{v}$ and $v_\emptyset^n$ for n sufficiently large. If the signal is irrelevant, the manager obtains $\int_0^1 \max\{P^{\sigma^E,\sigma^L}(s), v\}g(s)ds$, which is the same in the limit as $n \to \infty$ for $v_\emptyset^n$ as $\widetilde{v}$. If the signal is relevant, then since the market conjectures $v_\emptyset^n$ do not disclose and $\widetilde{v}$ does, and $\widetilde{v} < s^B$, we have that the payoff from nondisclosure if the signal is relevant is larger for the manager of type $\widetilde{v}$ than $v_\emptyset^n$. And by definition, the payoff from disclosure in the limit as $n \to \infty$ of $v_\emptyset^n$ is $\widetilde{v}$.

Putting this together, we conclude that the payoff from nondisclosure is strictly higher in the limit for $\widetilde{v}$ than $v_\emptyset^n$ managers, whereas the payoff from disclosure is the same. But since the $v_\emptyset^n$ managers do at least as well from not-disclosing as disclosing, the $\widetilde{v}$ manager must do strictly better. This suggests a profitable deviation, contradicting the hypothesis that σ^E was an equilibrium strategy. □

The previous argument shows that managers cannot disclose early with probability 1 if $v < s^B$. This does not yet imply that the expected payoff from not disclosing early can only equal $v - c^E$ if $v \geq s^B$; while the argument does show there cannot be any pure strategy equilibrium with this property, we must also consider the case of mixed strategies, where the manager is indifferent between disclosure decisions within some range.

Lemma A.7. *Suppose the manager is willing to disclose early given a value of v. Then $v > s^B$.*

Proof of Lemma A.7. Again we prove this result by contradiction. Note that the argument from Lemma A.6 implies that, if some $v < s^B$ obtains payoff $v - c^E$ in equilibrium, then this manager cannot disclose with probability 1; so, if managers are disclosing with positive probability, then they must be following a mixed strategy. If the market conjectures a manager with value v discloses with probability ϵ, then the expected payoff is:

$$\pi^{\sigma^E}(v,\epsilon) = q\frac{qv(1-p\epsilon) + (1-q)\int_0^1 v\sigma^E(v)g(v)dv}{q(1-p\epsilon) + (1-q)\int_0^1 \sigma^E(v)g(v)dv} + (1-q)\mathbb{E}[P^{\sigma^E}(s,\varnothing)].$$

Now, let v^* be the highest v such that this equals $v - c^E$ were $\rho(v) = 0$; again, since this is larger than $-c^E$, but since we have assume some manager is willing to disclose for $v < s^B$, we have that $v^* \in (0, s^B)$. Now, the value associated with $\rho(v) = 1$ and $\rho(v) = 0$ intersect at s^B, by the definition of s^B; for all $v < s^B$, since a pure strategy would (by the argument from Lemma A.6) imply a profitable deviation, we must have the value is equal to the expected payoff from non-disclosure in the range (v^*, s^B); for $v \geq s^B$, since the maximum possible payoff from nondisclosure increases at a rate less than 1, all such types must disclose with probability 1 in equilibrium.

So, this argument implies that:

- At v^*, the manager's payoff when the market conjectures the manager does not disclose is equal to $v^* - c^E$.
- At s^B, the manager's payoff when the market conjectures the manager discloses is equal to $s^B - c^E$, and is also equal to the expected price if the market were to conjecture the manager does not disclose.

Thus, we have argued that the function $\pi^{\sigma^E}(v, 0)$ intersects the line $v - c^E$ at two points; thus there must be some $\widetilde{v}$ where the slope of:

$$q\frac{qv + (1-q)\int_0^{v_L(v)} \widetilde{v}\sigma^E(\widetilde{v})g(\widetilde{v})dv}{q + (1-q)\int_0^{v_L(v)} \sigma^E(\widetilde{v})g(\widetilde{v})d\widetilde{v}},$$

is greater than 1; but we have argued previously (Lemma A.4) that this function (divided by q) has a slope less than 1 (so the slope when multiplying by q is even less). This contradiction establishes that if v is such that the manager's payoff from early disclosure is equal to the expected payoff from not disclosing early, it must be that $v \geq s^B$. $\square$

The previous two Lemmas imply that the manager can only possibly be willing to disclose early if v is such that $v > s^B$.

Lemma A.8. *Any equilibrium strategy σ^E is characterized by a threshold (possibly at the boundary) above which the manager discloses early and below which the manager does not disclose early.*

Proof of Lemma A.8. We only consider equilibria where early disclosure occurs with positive probability, since otherwise the threshold trivially is at 1. Clearly, if the manager discloses early, then the payoff is $v - c_E$, which increases in v at a rate of 1. We show that the payoff from non-disclosure increases at a rate less than 1 in v. Assume that v is such that $v - c_E < P^{\sigma_E,\sigma_L}(1, \varnothing)$, since otherwise the manager would always disclose early.

Let $\widetilde{v}$ be the infimum over manager values which are willing to disclose early. By the previous, we have that $\widetilde{v} > s^B$. We show that all values with $v > \widetilde{v}$ strictly prefer to disclose early. There are two cases to consider: (i) signal s is uninformative; (ii) signal s is informative.

Consider the first case. The manager's payoff, as a function of the signal s, is:

$$\max\{v - c^L, P^{\sigma^E,\sigma^L}(s, \varnothing)\}.$$

Since the signal is uninformative, $P^{\sigma^E,\sigma^L}(s, \varnothing)$ is independent of v, so that this expression is weakly increasing in v for all s; thus, $\int_0^1 \max\{v - c^L, P^{\sigma^E,\sigma^L}(s, \varnothing)\} g(s) ds$ is also increasing in v, but at a rate less than 1 (since the integral does not change in the event that $P^{\sigma^E,\sigma^L}(s) > v - c^L$); this involves the manager disclosing when the signal is below $w^L(v)$ and not when it is above $w^L(v)$. In particular, as long as $v - c_E < P^{\sigma_E,\sigma_L}(1, \varnothing)$, and since $v - c_E > v - c_L$ (since otherwise disclosing early is strictly dominated), we know that there are a range of signal realizations such that the manager would *not* disclose late.

Now consider the second case. Since $v > s^B$, we therefore have that, no matter what inference the market makes about the probability of relevance following signal v, the market price is lower than v itself. Indeed, at $v = s^B$, then the market price is constant in belief about relevance. Now, the change in the price following a *relevant signal* for a manager with type $\widetilde{v}$ versus type v is bounded by the change in the price if non-relevant signals are not-disclosed. Since the rate of change of the price for such signals in the auxiliary game is less than 1, we have that the manager's payoff increases at a rate less than 1 if they are in a region where they do not disclose late following relevant signals. If this condition does not hold, then the payoff from not disclosing early increases at a rate equal to exactly 1.

Putting this together, we see that the total rate of change in the payoff from not disclosing early is a convex combination of a function with slope at most 1 and a function with slope strictly less than 1. Importantly, since the weights on each of these terms themselves do not vary with the signal (since relevance is independent of v), we have that the overall rate of change in the payoff from not disclosing early is increasing at a rate strictly less than 1. It follows that if some manager with value v' weakly prefers to disclose early, then any manager with $v > v'$ strictly prefers to disclose early, and conversely if a manger with value v' weakly prefers to not disclose early, then any manager with value $v < v'$ strictly prefers to not disclose early. Thus, all equilibria σ^E are characterized by thresholds. □

A.3 Late Thresholds

Having showed that the early disclosure decision must be a threshold, we now show first, that the standard arguments from past work can be applied to characterize the late disclosure threshold, and second, that we can take the late disclosure threshold to be continuous in the external signal. We also show that the disclosure thresholds in the auxiliary game also determine the thresholds in the original game, with market beliefs about relevance "switching" at s^B.

We start by showing that the auxiliary game has well-behaved equilibrium disclosure strategies. Toward the first step, we follow the usual procedure of showing that higher types benefit from (late) disclosure more than lower types. Here, we use our previous result that the early disclosure equilibrium must be characterized by a threshold which we denote. We consider the expected payoff of the manager as a function of an arbitrary late-disclosure threshold, $\overline{v}$; without loss, we take this to be less than v^E, since the market infers that all managers with values $v > v^E$ disclosed early. Define:

$$H(s, \overline{v}, \epsilon) := \mathbb{E}[v|s, v \leq \overline{v}, \epsilon], \tag{8}$$

as the equilibrium nondisclosure payoff given a threshold in the auxiliary game (given ϵ), where we note that we do not need to consider the early disclosure strategy because $\overline{v} \leq v^E$.

Lemma A.9. *The function $H(s, \overline{v}, \epsilon, \sigma^E) - \overline{v}$ is decreasing.*

Proof of Lemma A.9. Note that:

$$H(s, \overline{v}, \epsilon) = \overbrace{s \Pr(\rho = R|s, v \leq \overline{v}, \epsilon)}^{(i)} + \overbrace{\mathbb{E}[v|\rho = N, s, v \leq \overline{v}, \epsilon] \Pr(\rho = N|s, v \leq \overline{v}, \epsilon)}^{(ii)} \tag{9}$$

by the Law of Iterated Expectations. Note that $H(s, \overline{v}, \epsilon)$ is differentiable in $\overline{v}$, since $\overline{v}$ only shows up as the right endpoint in the integrals which define these conditional expectations. We show that $\frac{d}{d\overline{v}} H(s, \overline{v}, \epsilon) < 1$.

Consider (ii): we show that it is increasing at a rate less than 1. First, write the expression as $\mathbb{E}[v\mathbb{1}_{\rho=N}|s, v \leq \overline{v}, \epsilon]$. Note that s only influences the probability that $\rho = N$, and in particular does not influence the distribution over v (since if $\rho = N$ then s conveys no information about v). Furthermore, ϵ has no direct influence on the manager's strategy on the event that $\rho = N$, by definition; adjusting this probability if necessary, we can drop this from the conditioning event. Furthermore, $\mathbb{E}[v\mathbb{1}_{\rho=N}|v \leq \overline{v}, \epsilon]$ increases at a rate less than $\mathbb{E}[v|v \leq \overline{v}, \epsilon]$; indeed, both $\mathbb{E}[v\mathbb{1}_{\rho=N}|v \leq \overline{v}, \epsilon]$ and $\mathbb{E}[v(1-\mathbb{1}_{\rho=N})|v \leq \overline{v}, \epsilon]$ are increasing in $\overline{v}$, and $\mathbb{E}[v|v \leq \overline{v}, \epsilon]$ is the sum of these two. Furthermore, by Lemma A.1, since g is log-concave, we have that $\mathbb{E}[v|v \leq \overline{v}]$ increases at a rate less than 1.

On the other hand, consider (i). We can write this term as $s\frac{q(1-p\varepsilon)}{q(1-p\varepsilon)+(1-q)((1-p)+p\int_0^{\overline{v}}\sigma^E(v)g(v)dv)}$. Upon inspection, this is *decreasing* in $\overline{v}$, since as $\overline{v}$ increases the denominator increases and the numerator is constant.

Putting this together, we have shown that $\frac{d}{d\overline{v}}H(s, \overline{v}, \epsilon, \sigma^E)$ is the sum of a term with slope less than 1 and a term which is non-increasing; thus, $\frac{d}{d\overline{v}}H(s, \overline{v}, \epsilon, \sigma^E) < 1$. □

Since Lemma A.2 states that any late-disclosure equilibrium must be characterized by a threshold (given s), Lemma A.9 implies that the equilibrium strategy in the auxiliary game is unique conditional on ϵ. Indeed, the fact that $H(s, \widehat{v}, \epsilon) - \widehat{v}$ is strictly decreasing in $\widehat{v}$ means that either (i) there exists a unique $\widehat{v}$ solving $H(s, \widehat{v}, \epsilon) - \widehat{v} = -c^L$, or (ii) $H(s, \widehat{v}, \epsilon) - \widehat{v} \geq -c^L$ for all $\widehat{v}$; in the latter case, no manager types disclose. Note that in particular we cannot have $H(s, 0, \epsilon) \leq -c^L$, since even if $c^L = 0$, we assume the manager is uninformed with positive probability, meaning that the left hand side is positive.

It remains to show that ϵ is pinned down in equilibrium, since these results leave open the possibility for multiplicity in the case that the market makes different conjectures about the manager's strategy regarding disclosure. In other words, the theorem requires us to find a unique equilibrium of the original game, and not the auxiliary game. To that end, define $\widehat{v}^L_\varepsilon$ as the unique second period threshold in the auxiliary game (i.e., when the market conjectures that relevant signals are disclosed by informed with probability ε). Recall that $v^L(s)$ is the late disclosure threshold of the original game.

Lemma A.10. *There exists some $\alpha \in [0, 1]$ such that $v^L(s) = \alpha\widehat{v}^L_1(s) + (1-\alpha)\widehat{v}^L_0(s)$ is an arbitrary threshold.*

Proof of Lemma A.10. Note that $P^{\sigma^E,\sigma^L}(s, \varnothing)$ must itself be a convex combination of $H(s, \overline{v}, 1)$ and $H(s, \overline{v}, 0)$, since it is the expectation over the event that the manager discloses a relevant signal, given all other parameters. Assume $H(s, v^L(s), 1) \geq H(s, v^L(s), 0)$; analogous arguments hold when this inequality is flipped. Then $H(s, v^L(s), 1, \sigma^E) \geq P^{\sigma^E,\sigma^L}(s, \varnothing) \geq H(s, v^L(s), 0)$. So if we have $v^L(s) - c^L > H(s, v^L(s), 1)$, then $v^L(s) - c^L > P^{\sigma^E,\sigma^L}(s, \varnothing)$, and if $v^L(s) - c^L < H(s, v^L(s), 1)$, then $v^L(s) - c^L < P^{\sigma^E,\sigma^L}(s, \varnothing)$. So, since the conditional expectations are all increasing in $v^L(s)$, if the manager is indifferent between late disclosure decision—i.e.,

$v^L(s) - c^L = P^{\sigma^E,\sigma^L}(s,\varnothing)$—then by this argument, $v^L(s)$ cannot be outside of the interval defined by $\widehat{v}_1^L(s)$ and $\widehat{v}_0^L(s)$. □

By Lemma A.10, any candidate threshold must be in between the "extreme" thresholds in the auxiliary game, $\widehat{v}_1^L(s)$ and $\widehat{v}_0^L(s)$, so for the moment we consider the behavior of these thresholds *as a function of* s. Note that it is immediate that $\widehat{v}_\epsilon^L(s)$ is increasing in s; indeed, this follows since $H(s,\widehat{v},\epsilon,\sigma^E) - \widehat{v}$ is increasing in both s and decreasing in $\widehat{v}$, so that if s increases $\widehat{v}$ must increase as well in order to make the manager indifferent between disclosure decisions. The next two Lemmas present some additional properties of these functions as well.

Lemma A.11. *The thresholds* $\widehat{v}_1^L(s)$ *and* $\widehat{v}_0^L(s)$ *are continuous in* s.

Proof of Lemma A.11. Consider $\widehat{v}_1^L(s)$, since the argument for $\widehat{v}_0^L(s)$ is analogous. Recall that $H(s,\widehat{v},1)$ is differentiable in $\widehat{v}$, since $\widehat{v}$ only appears as the endpoint of an integral in the expression defining it, and in particular Lemma A.9 shows that the slope increases at a rate less than 1. On the other hand, the threshold is defined implicitly as the solution to the equation $-c^L = H(s,\widehat{v},1) - \widehat{v}$. Since the derivative of the right hand side with respect to $\widehat{v}$ is non-zero, the implicit function theorem applies for any range of s with $\widehat{v}_1^L(s) \in (0,1)$. Noting that we can never have $\widehat{v}_1^L(s) = 0$ (since the payoff from nondisclosure is strictly positive in any threshold equilibrium, since $p < 1$), we point out that if in fact $\widehat{v}_1^L(s) = 1$, since this is constant at any s above which this holds, we still have continuity at any possible s. That is, the implicit function still applies in this case, so that $\widehat{v}_1^L(s)$ is continuous at the lowest s such that $\widehat{v}_1^L(s) = 1$; and, since it is constant above this s, we have it is continuous at all s as well. Thus, $\widehat{v}_1^L(s)$ is continuous in s, as claimed. □

Lemma A.12. *The thresholds* $\widehat{v}_1^L(s)$ *and* $\widehat{v}_0^L(s)$ *are increasing in* s *at a rate less than 1. Furthermore,* $\widehat{v}_1^L(s) - \widehat{v}_0^L(s)$ *is decreasing in* s.

Proof of Lemma A.12. Note that:

$$\frac{\partial}{\partial s}H(s,\widehat{v},0,\sigma^E) > \frac{\partial}{\partial s}H(s,\widehat{v},1,\sigma^E) \qquad \frac{\partial}{\partial \widehat{v}}H(s,\widehat{v},1,\sigma^E) > \frac{\partial}{\partial \widehat{v}}H(s,\widehat{v},0,\sigma^E).$$

Indeed, if s is never disclosed when it is relevant (i.e., $\epsilon = 0$), then upon seeing nondisclosure, the probability it is the true value is higher. Thus, an increase in s has more of an impact on the expectation, whereas an increase in $\widehat{v}$ has less of an impact. Thus, for a fixed change in s, in order to ensure that $H(s,\widehat{v},\epsilon,\sigma^E) - \widehat{v} = c^L$, the change in the left hand side is smaller when $\epsilon = 1$ than when $\epsilon = 0$, and furthermore that $\widehat{v}$ does not need to increase as much in order to ensure equality holds. Thus, $\widehat{v}_1^L(s)$ increases by less than $\widehat{v}_0^L(s)$. □

A.4 Uniqueness

We now put the above arguments together to present our results on uniqueness, starting with the late disclosure decision:

Lemma A.13. *The unique second period equilibrium strategy, conditional on the manager disclosing early above v^E, involves threshold $\widehat{v}_1^L(s)$ for $s > s^B$ and $\widehat{v}_0^L(s)$ for $s < s^B$.*

Proof of Lemma A.13. The proof essentially follows from putting together Lemmas A.10 and A.12. First, note that Lemma A.10 and the definition of v^B immediately implies that $\widehat{v}_{\sigma^E}^L(s^B) = v^B$. We consider two cases separately:

- Suppose that $s > s^B$. In this case, we note that by Lemma A.12, we have that $s > \widehat{v}_0^L(s) > \widehat{v}_1^L(s)$, and by Lemma A.10 the only candidate values for $v^{L\sigma^E}(s)$ satisfy $\widehat{v}_0^L(s) \geq v^{L\sigma^E}(s) \geq \widehat{v}_1^L(s)$. From inspection, we therefore see that, for any possible conjecture of the market regarding the manager's behavior following an informative signal, the signal s is above the market threshold. It follows that the only possible outcome in equilibrium is that the manager discloses informative signals. As a result, the unique equlibrium threshold is $v^{L\sigma^E}(s) = \widehat{v}_1^L(s)$.

- Suppose that $s > s^B$. Again by Lemma A.12, we have that $s < \widehat{v}_0^L(s) < \widehat{v}_1^L(s)$. Again from inspection, we see that for any possible conjecture of the market regarding the manager's behavior following an informative signal, the signal s is below the disclosure threshold. It follows that the only possible outcome in equilibrium is that the manager does not disclose informative signals. As a result, the unique equilibrium threshold is $v^{L\sigma^E}(s) = \widehat{v}_0^L(s)$.

Putting these observations together completes the proof. □

At this point, we have shown that we can restrict to equilibria where the first-period disclosure decision is characterized by a threshold, and furthermore, where the second-period disclosure threshold (function) is unique conditional on the first-period disclosure threshold. Lemma A.11 implies that this threshold is continuous in the external signal.

We now show that the nondisclosure payoff increases in the threshold at a rate less than 1, the last ingredient in our uniqueness argument.

Lemma A.14. *Suppose the market assumes that, when the manager is indifferent between disclosure decisions, disclosure occurs with probability $\widetilde{\epsilon}$. There exists at most one value v^E such that the manager is indifferent between disclosing early and not when the value is v^E, provided that $v^L(s)$ is itself an equilibrium disclosure threshold given the corresponding threshold v^E.*

Note that this shows the equilibrium is unique up to the choice of $\widetilde{\epsilon}$; this completes the proof of the theorem since we take equilibrium to involve $\widetilde{\epsilon} = 0$. See the discussion in the main text in Section 4.3 for a discussion of this assumption.

Proof of Lemma A.14. We first consider how the equilibrium price function changes in the *early* disclosure threshold. Using our assumption that $v^L(s)$ is itself an equilibrium disclosure decision, we can rewrite equation (7) with v^E explicitly written in. Let

$$r(s, \widehat{v}_0^L(s), v^E, \widetilde{\epsilon}) = \mathbb{1}_{s>\max\{\widehat{v}_0^L(s), v^E\}} + \widetilde{\epsilon}\mathbb{1}_{s=\max\{\widehat{v}_0^L(s), v^E\}},$$

refer to the probability that an informative signal would be disclosed. This yields:

$$0 = \widehat{v}_0^L(s) - c^L - \frac{qs(1-pr(s,\widehat{v}_0^L(s), v^E, \widetilde{\epsilon})) + (1-q)\left((1-p)\int_0^1 vg(v)dv + p\int_0^{\max\{\widehat{v}_0^L(s),v^E\}} vg(v)dv\right)}{q(1-r(s,\widehat{v}_0^L(s), v^E, \widetilde{\epsilon})) + (1-q)\left((1-p)\int_0^1 g(v)dv + p\int_0^{\max\{v^E,\widehat{v}_0^L(s)\}} g(v)dv\right)}. \tag{10}$$

Indeed, recall that this derivation held for an arbitrary first period strategy, and therefore holds assume the first-period strategy is used in the first period (which, unlike when this expression was first introduced, we have now shown must be the case). We consider two-cases separately:

- If $\widehat{v}_0^L(s)$ is such that $v^E > \widehat{v}_0^L(s)$, an increase in v^E has no impact on $\widehat{v}_0^L(s)$;
- If $\widehat{v}_0^L(s)$ is such that $v^E < \widehat{v}_0^L(s)$, then the increase in v^E increases $\widehat{v}_0^L(s)$ at a rate less than 1; the argument is identical to the proof from Lemma A.12, since v^E takes the same role as $\widehat{v}_0^L(s)$ in the ratio in equation (7).

Note that an identical argument applies to $\widehat{v}_1^L(s)$, meaning that this will also increase at a rate less than 1. So consider the equilibrium price function. Define v^B precisely as it was defined previously, namely as the intersection point of the thresholds in the two auxiliary games. If $\min\{\widehat{v}_1^L(s), \widehat{v}_0^L(s)\} < v^E$, then it is exactly the same as if $v^E = 1$. If $\min\{\widehat{v}_1^L(s), \widehat{v}_0^L(s)\} > v^E$, then it is equal to $\mathbb{E}[v|v \leq v^E]$ (in other words, if the intersection point is less than v^E, then v^E does not matter and we can set it equal to 1; if it is less than it, the price function is in a range where it is increasing at a rate less than 1, since the argument is identical to the late disclosure case).

With this in mind, we note the following:

- Consider the manager's expected payoff from not disclosing early if the second-period signal is informative. Note that, given $\widetilde{\epsilon}$, the payoff of the manager in this event is a convex combination, with weight $\widetilde{\epsilon}$, between $\lim_{v \to^- v^E} P^{\sigma^E,\sigma^L}(v,\varnothing)$ and $\lim_{v \to^+ v^E} P^{\sigma^E,\sigma^L}(v,\varnothing)$; but since both of these increase at a rate less than 1 (by the previous argument in this proof), the overall rate of change is less than 1.

- On the other hand, suppose the signal is uninformative. In this case, the fact that the manager's payoff increases at a rate less than 1 follows an identical argument from Lemma A.8; the manager's payoff given a signal s is $\max\{v^E - c^E, P^{\sigma^E,\sigma^L}(s,\varnothing)\}$; as long as $v^E - c^E$ is less than or equal to $P^{\sigma^E,\sigma^L}(1,\varnothing)\}$

  On the other hand, suppose that $v^E - c^E \geq P^{\sigma^E,\sigma^L}(1,\varnothing)$. Then in this case, since $s < 1$ with probability 1, the manager would strictly prefer to disclose. Therefore, the indifference condition can only be satisfied if v^E is such that $v^E - c^E < P^{\sigma^E,\sigma^L}(1,\varnothing)$.

  Putting these observations together, consider $\int_0^1 \max\{v^E - c^E, P^{\sigma^E,\sigma^L}(s,\varnothing)\}g(s)ds$. Since there is positive probability that the signal is such that $v^E - c^E < P^{\sigma^E,\sigma^L}(s,\varnothing)$, this cannot increase at a rate faster than 1.

Now crucially, the probability that the signal is informative is constant and independent of $\overline{v}$. Therefore, if $r_a(v^E)$ is the rate of change when the signal is relevant and $r_i(v^E)$ is the rate of change when the signal is not relevant, the total rate of change is $qr_a(v^E) + (1-q)r_i(v^E)$. Since $r^a(v^E) \leq 1$ and $r_i(v^E) < 1$ by the above arguments, provided that v^E is in a range such that indifference can possibly be satisfied (i.e., $v^E - c_4 < P^{\sigma^E,\sigma^L}(1,\varnothing)$. Therefore, the total payoff from nondisclosure cannot increase at a rate greater than or equal to 1 as v^E increases. Since the payoff in the event that the manager discloses early increases at a rate *equal* to 1, we have that there can only be one threshold v^E such that the manager is indifferent between disclosing early and not. □

A.5 Summary and Discussion

We now provide explicit proofs of the theorems, which essentially amount to tying together the above results.

Proof of Theorem 4. Note: given any arbitrary v^E, Lemma A.11 shows the implicit conditions for the thresholds imply that there exists an increasing and continuous function $\widehat{v}^L(s)$ such that the manager discloses when $v > \widehat{v}^L(s)$. The proof shows that this holds for all $s \neq s^B$, and so overall continuity follows from the observation that $\widehat{v}^L_1(s^B) = \widehat{v}^L_0(s^B)$, given in Lemma A.3. Thus, the late disclosure threshold, defined by the implicit condition, is continuous conditional on v^E. Existence then follows from noting that a manager at $v = 0$ would never prefer to disclose (and thus would never disclose early), together with continuity of the expected payoffs in v^E. (This leaves open the possibility that no manager would ever prefer to disclose early, in which case $v^E = 1$.)

□

Proof of Theorem 5. Uniqueness follows from Lemmas A.13 and A.14; note in particular that log-concavity of G implies log-concavity of distribution of the manager's value conditional on $v < v^E$, delivering uniqueness of the late disclosure thresholds; and furthermore, that given continuity of $v^L(s)$, we have the increasing difference property holds and delivers a unique value for v^E. □

Discussion. We conclude with a discussion of several properties that are of further interest, beyond the scope of the results at hand:

1. The results in Section A.2 show that *no* equilibrium—mixed or otherwise—can involve early disclosure below v^B.

2. If disclosure is only early, there exist multiple threshold equilibria that can arise by assuming different tiebreaking rules of the manager in the case of indifference. As discussed in the main text, this corresponds to the expected price intersecting the 45 degree line at different points between the left and right limit of the expected price function at the discontinuity. Our analysis implies all equilibria (including mixed) are of this form if disclosure is only early.

3. In the knife-edge cases where disclosure is either only early or only late and disclosure costs are zero—the cases discussed in depth in the main text—uniqueness can be obtained without log-concavity; the usual argument for uniqueness applies to the case of late disclosure, and for early disclosure this follows from the results in Section A.2, together with Lemmas A.4 and A.5.

4. We highlight that our setting involves disclosure dynamics that are generated by disclosure costs, a feature which in itself may be of independent interest. That is, for costs $c_L > c_E$, disclosure may occur both early and late. Most of the extra technical difficulties emerge because we accommodate for this possibility. We next mention some technical issues relevant more generally to future work on dynamic costly disclosure.

   We follow the standard approach of assuming log-concavity to obtain uniqueness—note that no such assumptions are needed for existence. To obtain unique late-disclosure thresholds with dynamics, we require log-concavity of G *conditional* on non-(early) disclosure. This step is non-trivial and our proof uses the restriction to continuous late-disclosure thresholds (since continuity implies that the slope is less than 1). In other words, given a threshold v^E, our proof shows uniqueness of a disclosure threshold $v^L(s)$ that is continuous and increasing in s—and visa versa. That said, we do not rule out exotic early-disclosure strategies which induce non-log-concave conditional distributions, supported by the manager switching between different $v^L(s)$ functions depending on the signal realization (with these switches generating discontinuities). This possibility strikes us as pathological, and we conjecture that it cannot emerge.

B Deterministic Correlation vs. Stochastic Relevance

In this supplemental analysis, we show that the key observations of our model—the kink in the market price at v^B and the downward drop in the price with early disclosure—cannot emerge if relevance is deterministic, at least with natural distributional assumptions. For instance, Acharya, DeMarzo and Kremer (2011) (in this section, ADK) assume, in their most general formulation, that the manager's value and the external signal are related by the equation:

$$v = \mu(s) + \sigma(s)z,$$

where z is independent of s (say, with distribution f) and, crucially, μ and σ are deterministic functions, with $\mu(s)$ strictly increasing. Assuming that v has the same distribution of z, the case of an irrelevant signal corresponds to $\sigma(s) = 1$ and $\mu(s) = 0$, whereas the case of a relevant signal corresponds to $\sigma(s) = 0$ and $\mu(s) = s$. In our setting, each of these occurs with an intermediate probability, which we denoted q.

When analyzing dynamics, ADK further take $\sigma(s)$ to be constant, equal to σ for all s. It is worth noting that, in the language of our model, threshold equilibria may not exist if relevance is arbitrary and non-constant in the manager's value—if, for instance, there is some $\widetilde{v}$ sufficiently close to 1 such that all signals above $\widetilde{v}$ are relevant with probability 1. In those cases, if disclosure is costly, such managers would rather *not disclose* (since the market infers v from the signal in the absence of disclosure). The same argument would hold as long as such signals are relevant with sufficiently high probability.

The reason such difficulties emerge is that, for early disclosure, the usual argument that equilibria must be of a threshold form now requires the *expectation* of the (future) non-disclosure price to be increasing by less than v. Whether this holds depends on (a) the market's (endogenously determined) updating rule, (b) the distribution over the external signal itself as a function of the firm value, and (c) the manager's subsequent strategy. With late disclosure (or without an external signal), investor belief following non-disclosure is constant, so such factors are not pertinent. These considerations emerge in other settings with external signals, and can generally complicate the existence of threshold equilibria (for instance, in the context of signalling as in Abrams, Libgober and List 2023). Thus, while threshold equilibria need not be guaranteed in general, our truth-or-noise specification does not have such tractability issues.

Furthermore, a standard argument shows that, if the distribution over z is log-concave and $\sigma(s)$ is constant, then the expected non-disclosure price must be increasing in s, given any disclosure strategy of the manager.[17] This assumption is satisfied by most simple distributions used in the literature (e.g., joint normality). Noting that the non-disclosure price is given by:

$$\mathbb{E}[v|s,\varnothing] = \frac{p\left(\int_{-\infty}^{\frac{\widehat{v}-\mu(s)}{\sigma}}(\mu(s)+\sigma z)f(z)dz\right)+(1-p)\mu(s)}{pF\left(\frac{\widehat{v}-\mu(s)}{\sigma}\right)+(1-p)},$$

we see that as long as $\mu'(s)$ and f are continuous, so is the slope of the price. Again, changing variables so that $\mu(s)=s$, we again remark that most common distributions are continuous on their support, ruling out any kink in the slope of the price, contrary to what we identify.

In our model, we instead take relevance (i.e., σ in the language of ADK) to be stochastic. The discontinuity we identify is a property of equilibrium, and emerges endogenously. Part of our contribution is to identify patterns of how relevance behaves in situations such as these—that is, the "grain of salt" updating pattern we identify suggests that it should be lower when signal realizations are higher. It seems less clear as to which assumptions would drive this exogenously, and we find it less plausible to depart from the literature by rationalizing the patterns (e.g., the kink in the price) using exotic distributions for the external signal.

C Imperfectly Informative (Relevant) Signals

We now discuss the sensitivity of our main predictions to the truth-or-noise structure of the external signal and show that, under a more general signal structure, our main results about *asymmetric* price reactions and *non-monotonicity* are maintained. Moreover, as long as the relevant signal is sufficiently informative, the price still drops sharply at v^E and, under

[17] This follows from Milgrom (1981). Changing variables so that $\mu(s)=s$, we can note $|\Pr(s \leq s^*|v) = \Pr(v-\sigma z \leq s^*|v) = \Pr(\frac{v-s^*}{\sigma} \leq z|v) = 1-F\left(\frac{v-s^*}{\sigma}\right)$. So, the density of s given v is $f\left(\frac{v-s^*}{\sigma}\right)/\sigma$; taking $v_1 > v_2$ and differentiating $f\left(\frac{v_1-s}{\sigma}\right)/f\left(\frac{v_2-s}{\sigma}\right)$, we see that it is increasing in s if f is log-concave, satisfying Milgrom (1981)'s strict MLRP condition.

certain conditions, this drop is *discontinuous*.

A more general formulation of the signal structure would allow for $s \sim F(\cdot \mid v, \rho)$. Given the prior over v, we could equivalently write $\widetilde{F}(\cdot \mid s, \rho)$ as the distribution over v conditional on s and ρ. Our main model corresponds to the case where $\rho = R$ implies F is a point mass at v and $\rho = N$ implies F is equivalent to G, independently of v.

Under the generalized structure, it is straightforward to show that the conclusion from Proposition 3 remains valid provided relevant signals are sufficiently informative and irrelevant signals are sufficiently uninformative:

Proposition C.1. *Suppose $\rho \in \{\rho_1, \rho_2\}$, with $\Pr(\rho = \rho_1) = q$, and parameters are such that, under truth-or-noise structure, there exists an early disclosure threshold less than 1. Let $\widetilde{F}^n(\cdot \mid s, \rho)$ be a sequence of information structures such that, as n increases, $\widetilde{F}^n(\cdot \mid s, \rho_1)$ converges weakly to δ_s and $\widetilde{F}^n(v \mid s, \rho_2)$ converges weakly to G.*[18] *For n sufficiently large, if v_n^E is an equilibrium threshold given information structure $\widetilde{F}^n(\cdot \mid \cdot, \cdot)$, then $v_n^E > v^B$.*

Proof. Let $P^{q,n}(s, \widehat{v})$ denote the equilibrium price with information structure $\widetilde{F}^n$ when the market assumes signals above $\widehat{v}$ are disclosed early. Note that $\Pr(\kappa, \rho \mid s, \varnothing, \widetilde{F}^n)$ converges to the value under truth-or-noise, as does the expectation of v conditional on these events, by the definition of weak convergence. Thus, $P^{q,n}(s, \widehat{v})$ converges to its value in the truth-or-noise information structure. Letting v^* denote the corresponding threshold under truth-or-noise, we note that by Proposition 3 this is above v^B. So, consider an arbitrary v', v'' such that $v^B < v' < v^* < v''$. We can find n sufficiently large such that if $\widehat{v} = v'$, a manager with value v' would strictly prefer to not disclose, and such that if $\widehat{v} = v''$, a manager with value v'' would strictly prefer to disclose. Since the price is continuous in $\widehat{v}$, and $\widehat{v}$ only appears as an endpoint in integrals defining the price, the intermediate value theorem implies that $v_n^E \in (v', v'')$, and in particular above v^B, as desired. □

Given this result, if there is early disclosure, it will be above v^B. This property drives the non-monotonicity result. To illustrate, suppose that irrelevant (ρ_2) signals are perfectly uninformative, but relevant signals (ρ_1) equal the true value, $s = v$, with probability $\alpha \in [0, 1]$, and equal the value with noise, $s = v + \varepsilon$ for $\varepsilon \sim U[-e, e]$ and e sufficiently small, with probability $1 - \alpha$:

1. First, consider the case where $\alpha = 0$ so that the distribution of relevant signals is smooth with $s = v + \varepsilon$ for $\varepsilon \sim U[-e, e]$ and e sufficiently small. Panels (a) and (b) of Figure 9 plot the price in this case, for a given conjectured threshold. We see that the price is still non-monotonic because of a price drop at the conjectured threshold.

[18] Here, δ_s is a Dirac measure with value s.

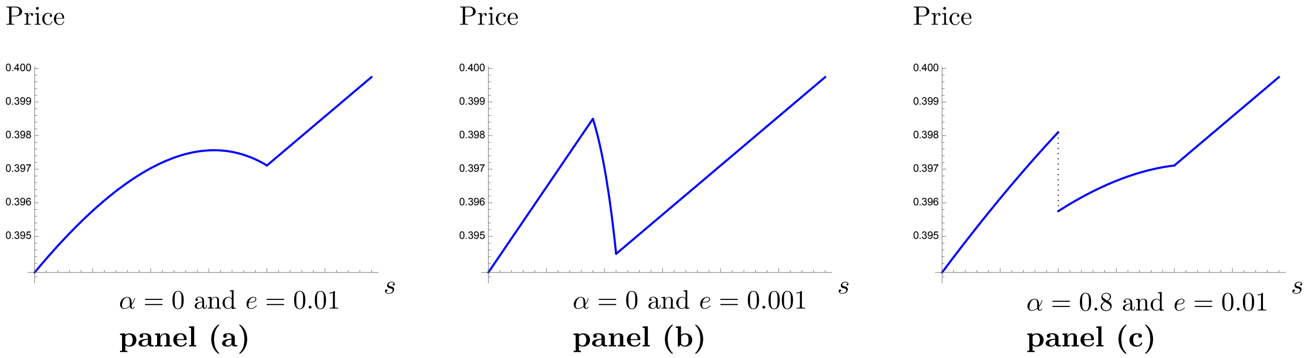

Figure 9: Nondisclosure price for given $\widehat{v}$ as a function of the external signal s
Numerical example with uniform distribution of v, $p = 0.6$, $q = 0.4$. Irrelevant (ρ_2) signals are uninformative. For relevant signals (ρ_1): $s = v$ with probability $\alpha \in [0, 1]$ and $s = v + \varepsilon$, for $\varepsilon \sim U[-e, e]$, with probability $1 - \alpha$. Here, $\widehat{v} = 0.41 > 0.39 = v^B$.

Smaller e implies a steeper price drop, and at $e \to 0$ approaches a discontinuous drop. Furthermore, the price is more sensitive to unfavorable external news.

2. Second, suppose now that $\alpha \in (0, 1)$ and sufficiently large, so that the signal structure maintains an atom. This case is illustrated in panel (c) of Figure 9. Notably, in this case, not only asymmetric price reaction and non-monotonicity, but also a discontinuity emerges, similar to our main model (but without the relevant signal being perfectly informative).

3. Lastly, if $\alpha = 1$, relevant signals perfectly reflect the true value as in the main model, and asymmetric price reactions, nonmonotonicity and discontinuity emerge as illustrated in panel (c) of Figure 3 in the main text.

More generally, given any specification of F, we can write:

$$v^\rho(s) = \mathbb{E}[v \mid s, \rho], \quad v^\rho(s \mid v \leq \widehat{v}) = \begin{cases} 0 & \text{if } \widetilde{F}(\widehat{v} \mid \rho, s) = 0, \\ \mathbb{E}[v \mid s, \rho, v \leq \widehat{v}] & \text{otherwise} \end{cases}$$

as the market expectation of the manager's value upon observing signal s. Note that $v^\rho(s) \geq v^\rho(s \mid v \leq \widehat{v})$, with strict inequality whenever ρ implies s does not perfectly reveal v. We also have:

$$\Pr(\rho, U \mid s, \varnothing) = \frac{\Pr(\varnothing, \rho, U \mid s)}{\Pr(\varnothing \mid s)} = \frac{(1-p)\Pr(\rho)}{(1-p) + p\sum_\rho \widetilde{F}(\widehat{v} \mid s, \rho)\Pr(\rho)}$$

$$\Pr(\rho, I \mid s, \varnothing) = \frac{\Pr(\varnothing, \rho, U \mid s)}{\Pr(\varnothing \mid s)} = \frac{p\Pr(\rho)\widetilde{F}(\widehat{v} \mid s, \rho)}{(1-p) + p\sum_\rho \widetilde{F}(\widehat{v} \mid s, \rho)\Pr(\rho)}.$$

Putting this together, the market price is:

$$P(s, \varnothing) = \sum_\rho v^\rho(s) \cdot \frac{(1-p)\Pr(\rho \mid s)}{(1-p) + p\sum_\rho \widetilde{F}(\widehat{v} \mid s, \rho)\Pr(\rho)} + v^\rho(s \mid v \leq \widehat{v}) \frac{p\Pr(\rho \mid s)\widetilde{F}(\widehat{v} \mid s, \rho)}{(1-p) + p\sum_\rho \widetilde{F}(\widehat{v} \mid s, \rho)\Pr(\rho \mid s)}.$$

It is easy to see that smoothness in $\widetilde{F}(\cdot \mid s, \rho)$ smooths prices, whereas jumps (caused by atoms) lead to discontinuity. As for non-monotonicity, note that $v^{\rho=N}(s) - v^{\rho=N}(s \mid v \leq \widehat{v})$ is constant in s, whereas $v^{\rho=R}(s) - v^{\rho=R}(s \mid v \leq \widehat{v})$ is 0 for $s \leq \widehat{v}$, with an upward jump at $s = \widehat{v}$. The non-monotonicity is driven by the property that $v^\rho(s) - v^\rho(s \mid v \leq \widehat{v})$ is flatter (and intermediate) when ρ reflects a less informative signal structure, but with a more informative signal structure is: (i) smaller when $s < \widehat{v}$ and (ii) larger for $s > \widehat{v}$. This is ultimately what drives the non-monotonicity, rather than the specifics of the signal structure itself. To emphasize, this property holds beyond truth-or noise, since when the signal structure is more informative, the conditioning event has less bite when $s < \widehat{v}$, but more bite when $s > \widehat{v}$.

References

[1] Abrams, E., Libgober, J., List, J. 2023. Research registries and the credibility crisis: an empirical and theoretical investigation. Working paper.

[2] Acharya, V., DeMarzo, P., Kremer, I. 2011. Endogenous information flows and the clustering of announcements. *American Economic Review* 101, 2955-2979.

[3] Aggarwal, R., Schirm, D. 1998. Asymmetric impact of trade balance news on asset prices. *Journal of International Financial Markets, Institutions and Money* 8, 83-100.

[4] Bagnoli, M., Bergstrom, T. 2005. Log-concave probability and its applications. *Economic Theory* 26(2), 445-469.

[5] Banerjee, S., Green, B. 2015. Signal or noise? Uncertainty and learning about whether other traders are informed. *Journal of Financial Economics* 117(2), 398-423.

[6] Barlevy, G., Veronesi, P. 2003. Rational panics and stock market crashes. *Journal of Economic Theory* 110, 234-263.

[7] Bond, P., Goldstein, I., Prescott, E. 2010. Market-based corrective actions. *Review of Financial Studies* 23 (2), 781–820

[8] Blot, C., Hubert, P., Labondance, F. 2020. The asymmetric effects of monetary policy on stock price bubbles. Working paper.

[9] Capkun, V., Lou, Y., Otto, C.A. and Wang, Y. 2022. Do firms respond to peer disclosures? Evidence from disclosures of clinical trial results. *The Accounting Review*, forthcoming

[10] Dye, R. 1985. Disclosure of non-proprietary information. *Journal of Accounting Research* 23, 123-145.

[11] Dye, R., Sridhar, S. 1995. Industry-wide disclosure dynamics. *Journal of Accounting Research*, 33, 157-174.

[12] Einhorn, E. 2018. Competing information sources. *The Accounting Review*, 93, 151-176.

[13] Frenkel, S., Guttman, I., Kremer, I. 2020. The effect of exogenous information on voluntary disclosure and market quality. *Journal of Financial Economics*, 138(1), 176-192.

[14] Frenkel, S., Guttman, I., Kremer, I. 2023. Disclosure with informed market. Work-in-progress.

[15] Gennotte, G., Leland, H. 1990. Market liquidity, hedging, and crashes. *American Economic Review* 30, 999-1021.

[16] Goldberg, Grisse, S. 2013. Time variation in asset price responses to macro announcements. *Federal Reserve Report, New York.*

[17] Grossman, S.J. 1981. The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics* 24, 461-484.

[18] Jung, W., Kwon, Y. 1988. Disclosure when the market is unsure of information endowment of managers. *Journal of Accounting Research* 26, 146-153.

[19] Kartik, N., Lee, F., Suen, W. 2019. A theorem on Bayesian updating and applications to communication games. Working paper.

[20] Kremer, I. and Schreiber, A. and Skrzypacz, A. 2021 Disclosing a random walk. Working paper.

[21] Menon, R. 2020. Voluntary disclosures when there is an option to delay disclosure. *Contemporary Accounting Research* 37, 829-856.

[22] Milgrom, P. 1981. Good news and bad news: representation theorems and applications. *The Bell Journal of Economics* 12, 380-391.

[23] Sletten, E. 2012. The effect of stock price on voluntary disclosure. *Review of Accounting Studies* 17.

[24] Veronesi, P. 1999. Stock market overreaction to bad news in good times: a rational expectations equilibrium model. *Review of Financial Studies* 12(5), 975-1007.

[25] Verrecchia, R. 1983. Discretionary disclosure. *Journal of Accounting and Economics* 5, 179-194.

[26] Xu, N., You, Y. 2023. Main Street's pain, Wall Street's gain. Working paper.